Analysis and Design of Natural and Threaded Space-Time Codes with Iterative Decoding

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Abstract

We consider code design of threaded and natural space-time codes [1, 2] and analyze their performance. We show by a counterexample that, for these codes, even if the binary rank criterion is not satisfyed, they demonstrate full diversity performance in the region of interest when iterative decoding is used. Hence, the relevant parameter for efficient code design is not the rank diversity. We conjecture that this parameter is the block-diversity. We provide simulation results, and, in order to reinforce our conjecture, we analyze the behavior of the iterative interference cancellation decoder by using density evolution with the Gaussian approximation.

1 Introduction

Multiple antennas are known to provide significant capacity gain in single-user wireless communications [3, 4]. Space-time codes (STC), attempt to perform close to capacity by multiplexing multiple signals and expoliting the diversity available in the fading channel. Design criteria have been proposed for STC to exploit full diversity [5] based on worst case pairwise error probability with maximum-likelihood decoding.

In this paper, we study threaded and natural STCs proposed in [1, 2] with iterative minimum mean squared error (MMSE) interference cancellation (IC) decoding. Assuming a genie aided decoder (i.e., a decoder whose observables for the transmitted symbols are computed assuming that all symbols of other antennas are known), we provide design guidelines based on the block diversity of single-input single-output codes for the BF channel [6]. For such randomlike codes, we show that traditional worst case design based on the rank diversity is not effective for typical frame error rate (FER) of interest. The behavior of iterative MMSE-IC decoding is studied with density evolution (DE) techniques, first introduced to analyze the performance and capacity of low-density parity-check codes (LDPC) [7] and used in [8] to analyze the performance of iterative multiuser detection in CDMA systems. The principle underlying DE techniques is that, for large codeword length, the pdf of the messages propagated by the iterative decoder at a given iteration, concentrates around the expected pdf resulting from a cycle-free graph. In his paper we provide a computationally efficient DE algorithm, based on the Gaussian approximation and improved bounding techniques [9, 10], in order to analyze iterative MMSE-IC decoding. Unlike previous works [11], that used DE for fully interleaved channels (infinite diversity order), we focus on the quasistatic channel, and we capture the diversity behavior of the code in the limit of infinite block-length.

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2 System Model

Generalized layered space-time codes. We consider a MIMO system with N_T transmit antennas and N_R receive antennas in a quasistatic fading environment. The received signal vector at time n, $\mathbf{y}_n \in \mathbb{C}^{N_R}$, is given by,

$$\mathbf{y}_n = \mathbf{H}\mathbf{x}_n + \mathbf{z}_n \quad n = 1, \ldots, N \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the fading matrix, $\mathbf{x}_n \in \mathcal{X}^{N_T}$ is the transmitted signal vector where $\mathcal{X} \subseteq \mathbb{C}$ denotes the signal constellation, $\mathbf{z} \in \mathbb{C}^{N_R}$ is the noise vector $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{I})$, and N is the codeword length. We consider the case where $\mathbf{H}$ is random, constant over N channel uses and perfecly known at the receiver. A generalized layered space-time code (STC) is defined by N_C component codes and a space-time modulation function $\mathcal{F} : \mathcal{C}_1 \times \cdots \times \mathcal{C}_{N_C} \to \mathcal{S} \subseteq \mathcal{X}^{N_T \times N}$, such that $\mathcal{F}(\mathbf{c}_1, \ldots, \mathbf{c}_{N_C}) = \mathbf{X}$, where $\mathcal{C}_c,\ c = 1, \ldots, N_C$ are the component codes, $\mathbf{c}_c \in \mathcal{C}_c,\ c = 1, \ldots, N_C$ are the component codewords, and $\mathbf{X} = [\mathbf{x}_1 \ldots \mathbf{x}_N]$ is the codeword matrix [2]. We study the case where $\mathcal{F}$ is obtained as the concatenation of a parsing function $\mathcal{P}$ that assigns coded symbols to transmit antennas, bit-interleaving and modulation over the signal set $\mathcal{X}$ according to a labeling rule $\mu : \mathbb{F}_2^M \to \mathcal{X}$, such that $\mu(b_1, \ldots, b_M) = x$, where $M = \log_2 |\mathcal{X}|$. When $\mathcal{C}_c,\ c = 1, \ldots, N_C$ are of rate r_c, the transmission rate of the resulting layered STC is $R = r_c N_T M$ bit/s/Hz.

Threaded and natural space-time codes. We consider the family of threaded space-time codes (TSTC), proposed in [1] as a new approach for layered STC. In TSTC, $N_C = N_T$ and the component codewords $\mathbf{c}_t \in \mathcal{C}_t$ are mapped onto the array $\mathbf{X}$

following the (layer, antenna, time) indexing triplet $\mathcal{P}(\ell, n) = (\ell, |n+\ell-1|_{N_T}, n)$ for $1 \leq n \leq N$ and $\ell = 1, \ldots, N_T$, thus having full spatial and temporal spans (see Fig. 1). TSTCs independently interleave coded symbols coming from the same layer and transmit antenna among each other.

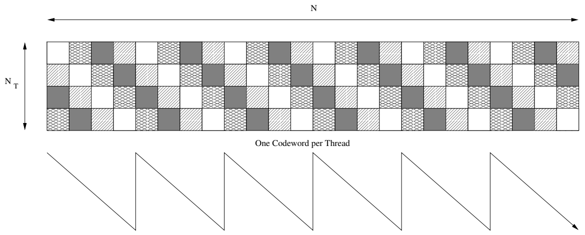

Figure 1: Threaded Space-Time Codes Layering.

Under the assumption that all interfering codewords are perfectly removed (genie aided decoder), each component codeword achieves the Matched Filter Bound (MFB), i.e. Maximal Ratio Combining (MRC) of the receive antennas. In such a case, the MIMO channel reduces to a set of non-interfering parallel channels. We define the *block diversity* of a STC $\mathcal{S}$ as the blockwise Hamming distance [6],

$$\delta_\beta = \min_{\mathbf{X}, \mathbf{X}' \in \mathcal{S}} |\{t \in [1, \ldots, N_T] \ : \ \mathbf{X}_t - \mathbf{X}'_t \neq 0\}|, \quad (2)$$

where $\mathbf{X}_t$ is the t-th row of $\mathbf{X}$, that is, the minimum number of nonzero rows of $\mathbf{X} - \mathbf{X}'$. Then, with a genie aided decoder, TSTC achieve diversity $\delta_\beta N_R$.

When component codes are trellis terminated convolutional codes of rate k/N_T, and generator matrices $\mathbf{G}_t \in \mathbb{F}_2^{N/N_T+\nu}$ $t = 1, \ldots, N_T$, where ν is the code memory, $\mathcal{F}_{TSTC}(\mathbf{c}_1, \ldots, \mathbf{c}_{N_T})$ maps for each component code, the output of generator $\mathbf{G}_t$ to antenna t.

Stacking construction [2]. Let $\mathbf{G}_1, \ldots, \mathbf{G}_{N_T}$ be binary matrices $\in \mathbb{F}_2^{K \times N}$, and consider the binary linear code of rate $K/(N_T N)$ generated by $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_{N_T}]$. Let the code words $\mathbf{c} = \mathbf{b}\mathbf{G}$ of $\mathcal{C}$, where $\mathbf{b} \in \mathbb{F}_2^K$, be parsed as

$$\mathbf{C} = \mathcal{F}(\mathbf{c}) = \begin{bmatrix} \mathbf{b}\mathbf{G}_1 \\ \mathbf{b}\mathbf{G}_2 \\ \vdots \\ \mathbf{b}\mathbf{G}_{N_T} \end{bmatrix}$$

Then, if for all $a_1, \ldots, a_{N_T} \in \mathbb{F}_2$ non all zero, the $K \times N$ matrix

$$\mathbf{M} = \bigoplus_{t=1}^{N_T} a_t \mathbf{G}_t$$

($\bigoplus$ indicates addition in the binary field $\mathbb{F}_2$) has rank K, then the BPSK STC obtained from $\mathcal{C}$ with the above parsing has full rank-diversity N_T (the condition is necessary and sufficient).

When $\mathcal{F} : \mathcal{C} \rightarrow \mathcal{S} \subseteq \mathcal{X}^{N_T \times N}$, such that $\mathcal{F}(\mathbf{c}) = \mathbf{X}$, anc $\mathcal{C}$ is a trellis terminated convolutional code of rate k/N_T and generator matrices $\mathbf{G}_{NSTC} = [\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_{N_T}]$, with $\mathbf{G}_t \in \mathbb{F}_2^{N+\nu}$, then, according to the stacking construction we have the natural STC (NSTC) where $\mathbf{b}\mathbf{G}_t$ is transmitted to antenna t. Maximum-likelihood (ML) decoding of NSTC may easily be implemented with the Viterbi algorithm [5]. By row-wise interleaving the NSTC array, we construct the interleaved NSTC (π-NSTC), with generator matrices $\mathbf{G}_{\pi-NSTC} = [\mathbf{G}_1\boldsymbol{\pi}_1, \mathbf{G}_2\boldsymbol{\pi}_2, \ldots, \mathbf{G}_{N_T}\boldsymbol{\pi}_{N_T}]$, and $\boldsymbol{\pi}_t \in \mathbb{Z}_+^{N \times N+\nu}$. In the limit, for large N, TSTC reduce to π-NSTC, except for the fact that in TSTC $\mathcal{C}_{TSTC} = \mathcal{C}_1 \times \cdots \times \mathcal{C}_{N_T}$ contains $N_T - 1$ trellis termination steps more than π-NSTC, thus serving as pilot states in the trellis of $\mathcal{C}_{TSTC}$. In addition, TSTC constrain the interleavers to be shorter, since each layer and antenna is to be interleaved independently. In Fig. 2, the π-NSTC transmission scheme is shown. Due to the presence of interleavers, ML decoding of π-NSTC is only possible by brute force computation, and we will proceed with iterative MMSE-IC decoding exactly as done for TSTC.

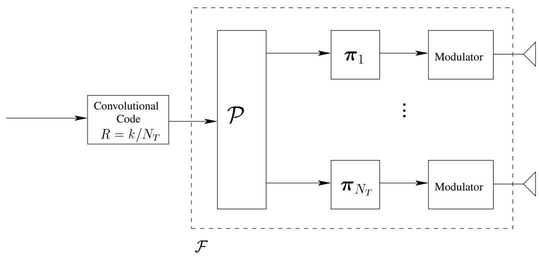

Figure 2: Transmission scheme for interleaved NSTC.

Extension to higher order modulations (M-PSK, M-QAM, M-APSK) is straigthforward. Since the modulation mapping μ is a bijective correspondence, the block diversity of the binary code is the same also for the modulated code. Then, π-NSTC with higher order modulation follow a multidimensional bit-interleaved coded modulation (BICM) [12] scheme.

Decoding. In order to emulate a genie aided decoder, a Unbiased Unconditional Minimum Mean Squared Error (MMSE) iterative interference canceller (IC) is proposed (see Fig. 3), where $\mathbf{f}_t^{(i)} = \alpha_t \sqrt{\gamma} \mathbf{R}^{-1} \mathbf{h}_t$ is the linear MMSE filter of antenna t, $\alpha_t = (\mathbf{h}_t^H \mathbf{R}^{-1} \mathbf{h}_t)^{-1}$ is the normalization constant, $\mathbf{R} = N_0 \mathbf{I} + \sum_{t=1}^{N_T} \mathbf{h}_t \mathbf{h}_t^H v_t^{(i-1)}$ is the covariance matrix of the input signal to the filter, and $v_t^{(i)} = E[|x_t - \hat{x}_t^{(i)}|^2]$ is the variance of the residual interference at antenna t at the i-th iteration ($v_t^{(0)} = 1$) (see [8] and references therein). Notice that $\mathbf{f}_t$ has to be computed once per transmit antenna and iteration, in contrast with the *conditional* MMSE which has to be computed once per symbol interval, transmit antenna and iteration [1]. The decision variable for antenna t at discrete time n and iteration i is,

$$z_{t,n}^{(i)} = \mathbf{f}_t^{(i)\ H} \big(\mathbf{y}_n - \sum_{t' \neq t}^{N_T} \mathbf{h}_{t'} \hat{x}_{t',n}^{(i-1)}\big) \quad (3)$$

where (dropping antenna and time indexes for simplicity), $\hat{x}^{(i)} = \sum_{x \in \mathcal{X}} x \prod_{m=1}^{M} \mathrm{P}_{ext}^{(i)}(\mu_m^{-1}(x))$, is the estimated symbol at the i-th iteration, where $\mu_m^{-1}(x) = a$ denotes that the m-th position of the bit label of x is equal to a. Then, the extrinsic log-likelihood ratio for the m-th bit of the binary mapping of the n-th symbol of antenna t at the i-th iteration is given by,

$$\mathrm{LLR}_{ext}^{(i)}(c_{t,n,m}|z_{t,n}^{(i)},\mathbf{H}) = \log \frac{\sum_{x\in\mathcal{X}_{m=0}} p(z_{t,n}^{(i)}|x,\mathbf{H}) \prod_{\substack{m'=1\\ m'\neq m}}^{M} \mathrm{P}_{ext}^{(i-1)}(c_{t,n,m'})}{\sum_{x\in\mathcal{X}_{m=1}} p(z_{t,n}^{(i)}|x,\mathbf{H}_b) \prod_{\substack{m'=1\\ m'\neq m}}^{M} \mathrm{P}_{ext}^{(i-1)}(c_{t,n,m'})} \quad (4)$$

where $\mathcal{X}_{m=a} = \{x \in \mathcal{X} \mid \mu_m^{-1}(x) = a\}$.

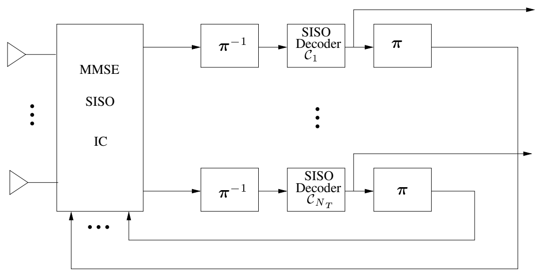

Figure 3: Iterative soft IC decoding scheme for TSTC.

3 Design Criteria and Examples

In [5], the authors proposed the well-known rank criterion for a space-time code to achieve the maximum available diversity in the system. We define the *rank diversity* of the STC $\mathcal{S}$ as

$$\delta_\rho = \min_{\mathbf{X},\mathbf{X}'\in\mathcal{S}} \mathrm{rank}(\mathbf{X}-\mathbf{X}'). \quad (5)$$

It is apparent that constructing codes maximizing δ_β is simpler than maximizing δ_ρ. Moreover, for codes over nonbinary constellations, δ_ρ may be very difficult to evaluate.

We can now look at TSTC or π-NSTC as binary space-time codes themselves, and simply apply the stacking construction theorem to check for its a priori diversity performance behavior. We found that for these codes, even if the stacking construction theorem is not satisfied, the frame error rate (FER) curves using sub-optimal iterative decoding show a full-diversity behavior in the region of practical interest, i.e., FER $\in [10^{-1}, 10^{-4}]$. Next we show by a counterexample that the rank criterion for TSTC and π-NSTC design is only a sufficient condition, not necessary, for the code to exploit all the available diversity in the region of practical interest. For the sake of notation simplicity and space limitation we will only report here the case of π-NSTC.

Counterexample: let $N_T = N_R = 4$ and consider the binary convolutional code $C = (5,7,7,7)_8$ of rate $r = 1/4$, and $\nu = 2$. As binary space-time code $\mathcal{C}$ mapped on BPSK (the corresponding spectral efficiency is $\eta = 1$ bit/s/Hz). We verify by applying the binary rank criterion that the NSTC obtanied from $\mathcal{C}$ has binary rank diversity $\delta_\rho = 2$. Clearly, $\delta_\beta = 4$. Now we apply the stacking construction to the π-NSTC resulting of $\mathcal{C}$, for block length $N = 128$. We generated 4 random permutations $\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_4$ and apply the theorem. We repeated the process for a large number of interleavers. None of them yielded a full-rank π-NSTC. However, as simulation results of Fig. 4 show, the performance of π-NSTC (and TSTC) over BPSK in the region of interest is parallel to that of $\mathcal{C}$ concatenated with a delay diversity (NSTC-DD) scheme (known to have $\delta_\rho = 4$). The few rank-deficient pairwise error events are not dominant, and they will only show their effect at extremely low FER far beyond practical interest, so at a certain point, TSTC and π-NSTC curves will cross that of NSTC-DD. Results are compared with the NSTC with ML decoding. We also report results obtained using the same code over QPSK and set-partitioning (SP) mapping ($\eta = 2$ bit/s/Hz) and 16-QAM with Gray mapping ($\eta = 4$ bit/s/Hz), showing the same performance slope with different spectral efficiencies η. Simulations use the iterative MMSE-IC approach described in the previous section. We run simulations with randomly generated interleavers fixed for the whole simulation. For the sake of completeness we also report the outage probability curves for the simulated spectral efficiencies.

Driven by these results, we conjecture that the relevant parameter to be maximized for efficient code design of TSTC and π-NSTC is the block diversity rather than the rank diversity. See also [13].

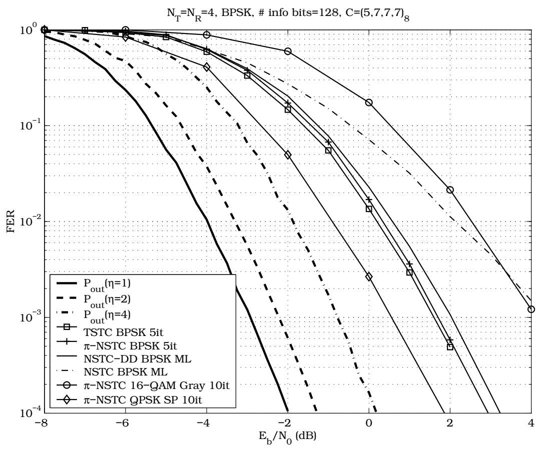

Figure 4: Performance of TSTC and NSTC.

4 Performance Analysis

In order to analyze the performance of TSTC and π-NSTC with MMSE-IC, and reinforce our conjecture, we resort to density evolution (DE) methods, first introduced to analyze the performance and capacity of low-density parity-check codes (LDPC) [7] and used in [8] to analyze the performance of iterative multiuser detection in CDMA systems. The main idea underlying DE techniques is that, as the codeword length tends to infinity, the pdf of the messages propagated by the iterative decoder at a given iteration i, concentrates around the expected pdf resulting from a cycle-free graph. In order to obtain a computationally efficient DE, we will use the Gaussian approximation (GA) and the tangential-sphere bound (TSB) to characterize the decoder (GA-TSB-DE). Notice that DE computes the bit error rate (BER) instead of the FER (that would always be equal to 1 for infinite codeword length).

For every realization of $\mathbf{H}$, the signal-to-noise ratios (SNR) at the output of the MMSE filter $\boldsymbol{\beta}^{(i)} = (\beta_1^{(i)}, \ldots, \beta_{N_T}^{(i)})$ at iteration i of the equivalent parallel channels are given by

$$\beta_t^{(i)} = \frac{E_s}{N_0|\mathbf{f}_t^{(i)}|^2 + \sum_{t'\neq t} |\mathbf{f}_t^{(i)}\mathbf{h}_{t'}|^2 v_t^{(i)}}. \tag{6}$$

The SNRs between the different antennas may be very different from each other. Recall that, as opposed to the CDMA case, large system arguments cannot be invoked, and therefore, convergence analysis must keep track of a vector of parameters.

We characterize the error probability given by the SISO decoder block as $\epsilon = f(\boldsymbol{\beta}^{(i)})$. In order to avoid exhaustive simulation, we resort to improved bounding techniques, i.e., the tangential-sphere bound, modified to handle a block fading channel [9, 10]. We define the multivariate weight enumeration function (MWEF) of $\mathcal{C}$, as the number of pairwise error events with output Hamming weights per antenna $w_1, \ldots, w_{N_T}$, $A_{w_1,\ldots,w_{N_T}}$.

For a given $\boldsymbol{\beta}^{(i)}$ and BPSK modulation, the squared Euclidean distance (SED) of a pairwise error event of output Hamming weights per antenna $w_1, \ldots, w_{N_T}$ is $d^2 = 4E_s \sum_{t=1}^{N_T} w_t \beta_t^{(i)}$. We then define the Euclidean distance Spectrum (EDS) of $\mathcal{C}$, A_d for $d = d_{min}, \ldots, d_{max}$ as the number of pairwise error events with Euclidean distance d.

In order to model the error rate fed back from the decoder at each antenna and iteration, we compute the symbol error rate (SER) per block, we define $A^t_{w_1,\ldots,w_{N_T}} = \frac{w_t}{N} A_{w_1,\ldots,w_{N_T}}$. Then, the tangential-sphere bound on the SER of antenna t at the i-th iteration is given by [10],

$$\begin{aligned}\epsilon_s^t(i) \leq \int_{-\infty}^{+\infty} \frac{dz_1}{\sqrt{2\pi\sigma^2}} e^{-z_1^2/2\sigma^2} \Bigg\{ 1 - \bar{\Gamma}\left(\frac{L-1}{2}, \frac{r_{z_1}}{2\sigma^2}\right) + \\ + \sum_{d:\, d/2<\alpha_d} A_d^t \,\bar{\Gamma}\left(\frac{L-2}{2}, \frac{r_{z_1}^2 - \beta_d(z_1)^2}{2\sigma^2}\right). \\ \cdot \left[\mathrm{Q}\left(\frac{\beta_d(z_1)}{\sigma}\right) - \mathrm{Q}\left(\frac{r_{z_1}}{\sigma}\right)\right] \Bigg\}\end{aligned} \tag{7}$$

where A_d^t is the EDS from $A^t_{w_1,\ldots,w_{N_T}}$ and $\boldsymbol{\beta}^{(i)}$, $L = N_T N$ is the codeword length, $\bar{\Gamma}(a,x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} dt$ is the normalized incomplete gamma function and $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt$ is the gamma function, $\mathrm{Q}(x) = 1/\sqrt{2\pi} \int_x^{\infty} e^{-(t^2/2)} dt$ is the Gaussian tail function, $\sigma^2 = N_0/2$, $r_{z_1} = r(1 - z_1/R)$, $\beta_d(z_1) = \frac{r_{z_1}}{\sqrt{1 - d^2/R^2}} \frac{d}{2r}$, $\alpha_d = r\sqrt{1 - d^2/R^2}$, $R^2 = E_s N \sum_{t=1}^{N_T} \beta_t^{(i)}$ and r is the solution of

$$\sum_{d:\, d/2<\alpha_d} A_d^t \int_0^{\theta_k} \sin^{N-3}\phi\, d\phi = \frac{\sqrt{\pi}\,\Gamma\left(\frac{L-2}{2}\right)}{\Gamma\left(\frac{L-1}{2}\right)} \tag{8}$$

with $\theta_k = \cos^{-1}\left(\frac{d}{2r} \frac{1}{\sqrt{1 - d^2/R^2}}\right)$.

In order to avoid the computation of the LLRs density function at the decoder's output, we use the so-called Gaussian Approximation (GA), and we consider that the a posteriori $\mathrm{LLR}^t_{app} \sim \mathcal{N}(\mu^t_{app}, 2\mu^t_{app})$. Then, $\epsilon_s = \Pr(\mathrm{LLR}_{app} < 0) = \mathrm{Q}\left(\sqrt{\frac{\mu_{app}}{2}}\right)$, which gives that $\mu^t_{app} = 2[\mathrm{Q}^{-1}(\epsilon^t_s)]^2$. We model the extrinsic $\mathrm{LLR}^t_{ext} = \mathrm{LLR}^t_{app} - \mathrm{LLR}^t_{in} \sim \mathcal{N}(\mu^t_{ext}, 2\mu^t_{ext})$, with $\mu^t_{ext} = \mu^t_{app} - \mu^t_{in}$, where $\mathrm{LLR}^t_{in} \sim \mathcal{N}(\mu^t_{in}, 2\mu^t_{in})$, is the LLR at the decoder input, with $\mu^t_{in} = 4\beta_t$. Therefore, the residual interference variances for the next iteration are $v_t^{(i+1)} = 1 - E[\tanh^2(\mathrm{LLR}^{t,(i)}_{ext}/2)]$ [1]. The bit error rate (BER) $\epsilon_b(i)$ at each iteration can be bounded using the same technique by replacing A_d^t by $A'_d = \sum_h \frac{h}{K} A_{h,d}$ in (7) and (8), where $A_{h,d}$ is the number of pairwise error events with input Hamming weight h and euclidean distance d.

5 Numerical examples

In this section we report some numerical examples of the DE algorithm presented in the previous section. Fig. 5 shows the evolution with the iterations of the BER, for a given channel matrix $\mathbf{H}$ for $E_b/N_0 = 0$dB. It can be observed that, for this given realization of $\mathbf{H}$ and SNR, there is a very good correspondence between the GA-TSB-DE method and the simulation for large codeword length (K=100000) and not-so-large codeword length (K=128). Moreover we also see that the MFB is achieved after 3 iterations. For the sake of completeness we also report the BER of the ML decoder of NSTC. Notice that, for lower SNR or a *bad* $\mathbf{H}$, the matching between simulation and GA-TSB-DE may not be so good, making the code work at the region where the TSB is not tight. In Fig.6 we show the evolution chart of the residual interference variance vector. As observed, there is a fixed point at a very low value, which indicates that the MFB will be closely approached. Notice that, due to the GA and TSB on SER per block and BER, GA-TSB-DE does not yield any upper nor lower bound on the BER performance, but only an approximation. We have computed the truncated MWEF with a modified version of the algorithm in [14], in order to handle multivariate simple error events[2].

We finally show in Fig. 7 the Montecarlo average BER and FER performance predicted by the GA-TSB-DE compared to simulation. Notice that, strictly speaking, DE methods yield FER $= 1$. Therefore, in order to estimate the FER, which is the relevant performance measure for the case of our study, we have computed the TSB for the FER by using the truncated MWEF $A_{w_1,\ldots,w_{N_T}} = K A^{\mathrm{simple}}_{w_1,\ldots,w_{N_T}}$ for K = 128. As we observe from the figure, there is a very good matching between simulation and the prediction by GA-TSB-DE. Indeed, GA-TSB-DE predicts very well the diversity gain of the iterative MMSE-IC decoder and provides good performance estimation. As mentioned in a previous section, since π-NSTC are not full

[1]This expectation can be easily computed using the Gauss-Hermite quadrature rule.

[2]Notice that computing only simple error events is needed for convolutional codes, and that $A_{w_1,\ldots,w_{N_T}} = K A^{\mathrm{simple}}_{w_1,\ldots,w_{N_T}}$ counts for its shifts, where $A^{\mathrm{simple}}_{w_1,\ldots,w_{N_T}}$ is the MWEF for simple error events.

rank, its performance will cross that predicted by GA-TSB-DE (infinite length π-NSTC are full rank by definition), and will show a diversity floor effect, similar to that of turbo-codes.

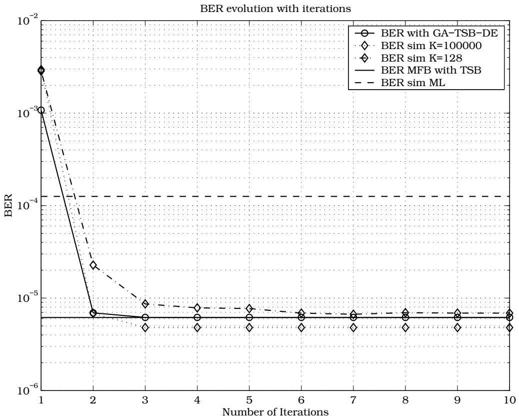

Figure 5: Snapshot for fixed $\mathbf{H}$ of BER vs. iterations for $E_b/N_0 = 0\text{dB}$ and $(5,7,7,7)_8$ CC.

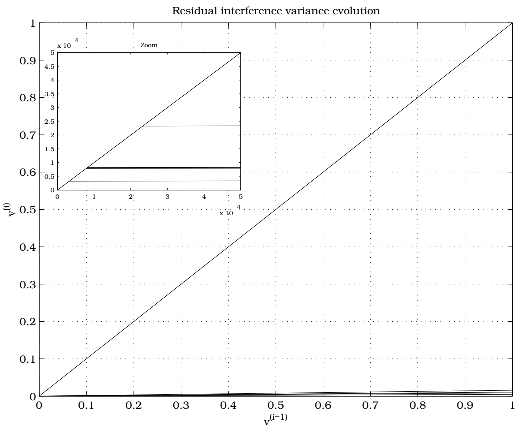

Figure 6: Snapshot for fixed $\mathbf{H}$ of the residual interference variance evolution for $E_b/N_0 = 0\text{dB}$ and $(5,7,7,7)_8$ code.

6 Conclusions

In this paper we have studied TSTCs and NSTCs with iterative decoding over a MIMO quasistatic fading channel. We have shown that, even when TSTCs and π-NSTCs are rank deficient, they show full diversity performance in the region of practical interest. We have conjectured that the relevant parameter for efficient code design is the block diversity. In order to support our conjecture, we have proposed a new method to analyze the behavior of the iterative MMSE-IC decoder by using computationally efficient density evolution techniques with the Gaussian approximation and improved bounding techniques.

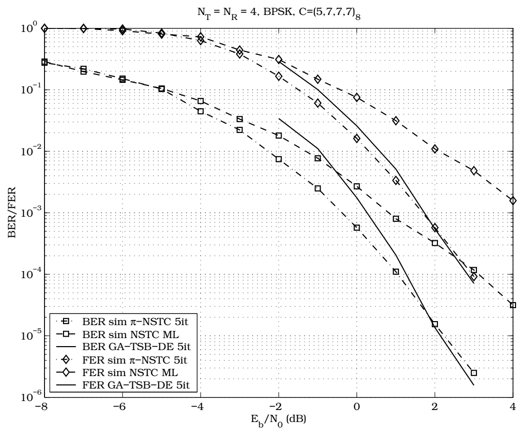

Figure 7: BER/FER of π-NSTC and NSTC $(5,7,7,7)_8$ code.

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