Design of Space-Time Bit-Interleaved Coded Modulation for Block Fading Channels with Iterative Decoding

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Abstract — **Bit-interleaved coded modulation space-time codes over block-fading channels with iterative decoding are investigated. We study the cases with independent and correlated fading blocks. We propose code construction based on the algebraic properties of the binary code, that lead to smart greedy space-time codes. We show the relevance of interleaver design in the diversity performance of the proposed codes. Belief propagation iterative decoding and low-complexity interference cancellation approximations are examined. We illustrate by simulation the interaction between coding rate, modulation signal set, binary labeling and decoding algorithm.**

I. Introduction

Multiple antennas are known to provide significant capacity gain in single-user wireless communications [1, 2]. In block-fading (BF) multiple-input multiple-output (MIMO) channels, space-time codes (STC) attempt to perform close to outage capacity by exploiting the diversity available in the fading channel. Coding for MIMO BF channels has been studied [3]. The same principles apply to orthogonal frequency division multiplexing (OFDM), where the fading blocks are identified with the frequency-flat subchannels obtained in the frequency domain [4, 5].

Bit-interleaved coded modulation (BICM) was proposed for single antenna systems in [6]. Classical BICM used Viterbi decoding. Iterative decoding of BICM was considered in [7] showing remarkable performance gain for some binary labeling rules. BICM STCs with either Viterbi or iterative decoding have been considered in [8, 9, 10] (and references therein) for time-selective channels. BICM STCs with Viterbi decoding are proposed in [11] for frequency selective OFDM channels. For BICM STCs, diversity is directly exploited by an error control code.

In this paper, we study pragmatic BICM STCs based on a sub-block partition of a binary code and the application of a bit-interleaved modulator block-by-block. We examine iterative belief propagation (BP) decoding and interference cancellation (IC) with linear filtering approximations (see [12, 13] and references therein in the context of CDMA and MIMO STC respectively). Assuming a genie aided decoder (i.e., a decoder whose observables for the transmitted symbols are computed assuming that all symbols of other antennas are known), we provide design guidelines based on the block diversity of single-input single-output codes for the BF channel [14]. For such randomlike codes, we show that traditional worst case design based on the rank diversity is not effective for typical frame error rate (FER) of interest. Using these guidelines, we construct smart-greedy codes for the BF with independent and correlated blocks. We study by simulation the interplay between coding rate, modulation signal set, binary labeling and bitwise soft output demodulator such that the genie aided assumption holds.

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II. System model

We consider a MIMO system with N_T transmit antennas and N_R receive antennas in a block-fading environment with N_B blocks. The received signal matrix corresponding to the b-th block $\mathbf{Y}_b \in \mathbb{C}^{N_R \times L_B}$ is,

$$\mathbf{Y}_b = \sqrt{\gamma}\mathbf{H}_b\mathbf{X}_b + \mathbf{Z}_b, \quad b = 1, ..., N_B, \tag{1}$$

where $\mathbf{H}_b = [\mathbf{h}_{1,b} \dots \mathbf{h}_{N_T,b}] \in \mathbb{C}^{N_R \times N_T}$, is the fading channel matrix over block b, with $\mathbf{H} = [\mathbf{H}_1 \cdots \mathbf{H}_{N_B}]$, $\mathbf{X}_b \in \mathcal{X}^{N_T \times L_B}$ is the b-th block of the transmitted signal matrix $\mathbf{X} = [\mathbf{X}_1 \cdots \mathbf{X}_{N_B}] \in \mathcal{X}^{N_T \times L}$ with $L = N_B L_B$, $\mathbf{Z}_b \in \mathbb{C}^{N_R \times L_B}$ is a matrix of noise samples i.i.d. $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$, and γ is the average signal to noise ratio (SNR) per transmit antenna. The channel is normalized such that $\frac{1}{N_T N_R}\text{trace}(E[\mathbf{H}_b\mathbf{H}_b^H]) = 1$, for $1 \leq b \leq N_B$, so the average SNR per receive antenna is $N_T\gamma$. The elements of $\mathbf{H}_b$ are assumed to be i.i.d. circularly symmetric Gaussian random variables (Rayleigh fading).

This model can represent general time/frequency selective fading channels where the blocks are identified with time/frequency slots. Fading blocks may be either independent or correlated. For the case of independent blocks the two extreme cases of quasistatic and fully interleaved channels can be obtained from (1) by letting $N_B = 1$ and $N_B = L$ respectively. We will also consider the case of slowly varying frequency-selective channels where the blocks are identified with the subcarriers of an OFDM system with N_c subcarriers. We assume perfect channel state information (CSI) at the receiver (CSIR).

We consider STCs defined by a binary block code $\mathcal{C} \subseteq \mathbb{F}_2^N$ of length N and rate r and a spatial modulation function $\mathcal{F} : \mathcal{C} \to \mathcal{S} \subseteq \mathcal{X}^{N_T \times L}$, such that $\mathcal{F}(\mathbf{c}) = \mathbf{X}$. We study the case where $\mathcal{F}$ is obtained as the concatenation of a block/antenna parsing function $\mathcal{P} : \mathbb{Z}_+ \to \mathbb{Z}_+^2$ such that $\mathcal{P}(n) = (t, \ell), 1 \leq n \leq N$, $1 \leq t \leq N_T$, $1 \leq \ell \leq LM$ that partitions a codeword $\mathbf{c} \in \mathcal{C}$ into sub-blocks, and blockwise bit-interleaving and modulation over the signal set $\mathcal{X}$ according to a labeling rule $\mu : \mathbb{F}_2^M \to \mathcal{X}$, such that $\mu(b_1, \dots, b_M) = x$, where $M = \log_2|\mathcal{X}|$ (see Figure 1). In this case, $N = N_T LM$. The transmission rate of the resulting STC is $R = rN_T M$ bit/s/Hz.

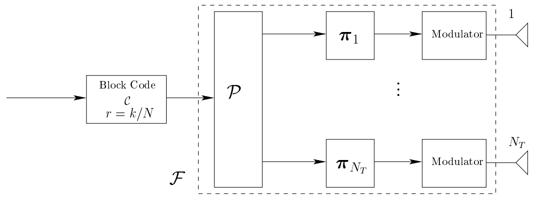

Fig. 1: Transmission scheme of BICM STC.

III. Conventional code design and motivation

In [15], criteria for STC design were derived for the quasistatic and fully interleaved channels. They considered the union bound and Pairwise Error Probability (PEP) for maximum likelihood (ML) decoding assuming perfect CSIR. For the BF channel, and assuming i.i.d. Rayleigh fading, the PEP between code words $\mathbf{X}$ and $\mathbf{X}'$ can be upperbounded by,

$$\begin{aligned} P(\mathbf{X} \to \mathbf{X}') &\leq \prod_{b=1}^{N_B} \prod_{t=1}^{N_T} \left(1 + \frac{\gamma}{4}\lambda_{t,b}\right)^{-N_R} \\ &\leq \left(1 + \frac{\gamma}{4}\sum_{t,b}\lambda_{t,b} + \left(\frac{\gamma}{4}\right)^{\rho} \prod_{(t,b):\lambda_{t,b}>0} \lambda_{t,b}\right)^{-N_R} \quad (2) \end{aligned}$$

where $\mathbf{D}_b = \mathbf{X}_b - \mathbf{X}'_b$ is the b-th block difference array and $\lambda_{t,b}$ is the t-th eigenvalue of $\mathbf{D}_b\mathbf{D}_b^H$. From the above bound, the standard space-time coding (STC) design criteria are found to be: 1) The rank-sum criterion: maximize $\rho = \sum_{b=1}^{N_B} \rho_b$, where $\rho_b = \text{rank}(\mathbf{D}_b)$, over all pairs $\mathbf{X} \neq \mathbf{X}'$, 2) the eigenvalue product criterion: maximize $\prod_{(t,b):\lambda_{t,b}>0} \lambda_{t,b}$, for the pairs $\mathbf{X} \neq \mathbf{X}'$ achieving min ρ, and 3) the eigenvalue sum criterion: maximize $\sum_{t,b} \lambda_{t,b}$, over all pairs $\mathbf{X} \neq \mathbf{X}'$ [3]. The eigenvalue sum criterion, also known as "trace" criterion ($\sum_{t,b} \lambda_{t,b} = \sum_{b=1}^{N_B} \text{trace}(\mathbf{D}_b\mathbf{D}_b^H)$) coincides with the standard *minimum squared Euclidean distance* criterion between codewords vec($\mathbf{X}$) and vec($\mathbf{X}'$), and is expected to dominate the upper bound at low-SNR (high FER). The rank and eigenvalue product criteria are expected to dominate the bound at high-SNR (low FER). The difficulty in constructing codes satisfying the above criteria is that the rank diversity is hard to evaluate and it is hardly related to the algebraic properties of the code, with the exception of binary codes [16].

In practical data transmission systems, an upper layer automatic repeat request (ARQ) protocol is to be considered. It retransmits frames in error until they are successfully decoded. For a code of rate R, the average throughput is, $\eta = R(1-P_F)$, where P_F is the FER. A much more meaningful criterion for data transmission would be to maximize η. It turns out that η is maximized for fairly large coding rate, implying large FER (see [17] and references therein). For example, assuming a Gaussian i.i.d. input distribution ideal random codes and asymptotically large N, $P_F = P(I(\mathbf{H}) \leq R)$, where $I(\mathbf{H})$ is the mutual information between $\mathbf{X}$ and $\mathbf{Y}$ for a given realization of the sequence $\mathbf{H}$, for a given input distribution. As we see from Figure 2, for fixed SNR, the FER corresponding to the maximum throughput is very large $\sim 10^{-1}$. Therefore, in order to maximize our throughput, we would be interested in using the multiple antennas for maximizing the rate and consequently keep high FER instead of minimizing the FER by transmitting at lower rates. Driven by this considerations, we conclude that the region of interest is not that of asymptotically large SNR.

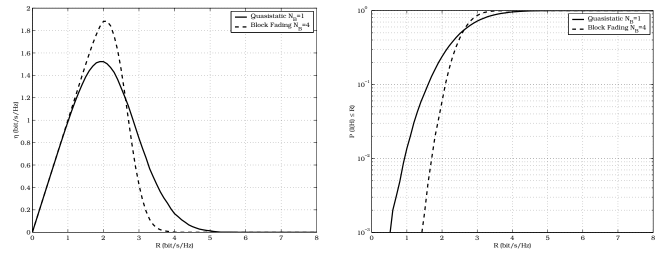

Fig. 2: Throughput of ARQ and Outage Probability for Gaussian inputs, $N_T = N_R = 2$ and $N_B = 1, 4$ and SNR= 3 dB.

IV. Code Design

In this section we first introduce code design criteria for the case of independent blocks. Based on that, and some a priori knowledge of the correlation of the channel, we propose pragmatic design criteria for block correlated BF channels.

BF Channel with Independent Blocks

We consider a genie aided decoder that produces observables of the transmitted symbols of one antenna, assuming that symbols from all other antennas are known[1]. In this way, the channel decomposes into an equivalent single-input single-output BF channel with $B = N_T N_B$ blocks,

$$\mathbf{r}_{t,b} = \sqrt{\gamma}|\mathbf{h}_{t,b}|\mathbf{x}_{t,b} + \boldsymbol{\nu}_{t,b}\ , \quad 1 \leq t \leq N_T, 1 \leq b \leq N_B, \quad (3)$$

where $\mathbf{r}_{t,b}$ is the received sequence of symbols corresponding to antenna t and block b, $\mathbf{x}_{t,b}$ is the transmitted sequence of symbols over antenna t and block b and $\boldsymbol{\nu}_{t,b}$ is the corresponding sequence of noise samples $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$.

We define the *block diversity* of a STC $\mathcal{S}$ mapped onto N_B blocks as the blockwise Hamming distance,

$$\delta_\beta = \min_{\mathbf{X},\mathbf{X}' \in \mathcal{S}} |\{(t, b)\ :\ \mathbf{x}_{t,b} - \mathbf{x}'_{t,b} \neq 0\}|, \quad (4)$$

for $t \in [1, \ldots, N_T]$, $b \in [1, \ldots, N_B]$. Then, with a genie aided decoder we *can* achieve diversity $\delta_\beta N_R$ [14]. Notice that applying BICM within a block, preserves the block diversity of the binary code, since the binary labeling rule μ is a bijective correspondence. For a code $\mathcal{C}$ of rate r over $\mathbb{F}_2$ mapped over B independent blocks, a fundamental upper bound on δ_β is provided by the Singleton bound (SB), $\delta_\beta \leq 1 + \lfloor B\,(1 - r) \rfloor$. Consequently, in our design we will search for codes maximizing δ_β, i.e., SB-achieving for all values of B (see Figure 3).

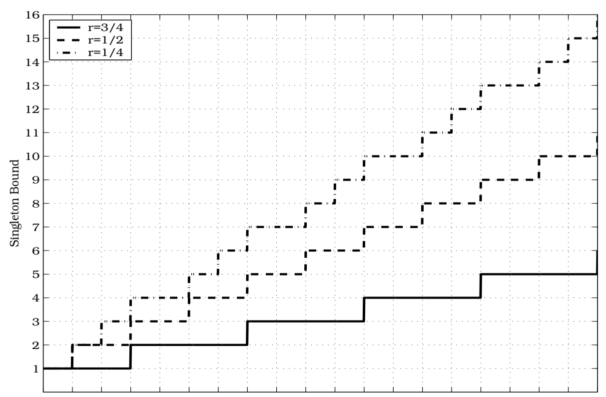

Fig. 3: Singleton Bound for $r = \frac{3}{4}$, $r = \frac{1}{2}$, $r = \frac{1}{4}$ binary codes.

[1] The reader will notice the analogy with the case of decision feedback equalization for frequency selective channels, where correct feedback is assumed to design the equalizer filters.

From now on we will restrict our attention to cyclically interleaved convolutional codes of rate $r = K/B$ [2], for which $\mathcal{P}$ is such that for a given hyper-trellis step, the b-th coded symbol is transmitted over the b-th fading block.

BICM STCs designed with the block-diversity criterion are not necessarily full rank, i.e., they do not achieve, in general, full diversity [18]. Nevertheless, we argue that rank deficient error events will only show their effect at SNRs well-above those of interest. This fact is confirmed by the simulations presented in section VI.

The proposed design criteria is a generalization of a number of works [13, 8, 11] (and references therein) where coded symbols from a convolutional code of rate $r = 1/N_T$ from the first generator are assigned to the first antenna and so forth [3]. The generalized layered architecture proposed in [13], where each layer is designed as a full-rank STC itself, assuming that all other layers can be removed, is inherently based on the block diversity criterion and, as said above, the codes constructed in this way are not necessarily full-rank.

In a fully interleaved channel every coded symbol undergoes independent fading and $\delta_\beta = d^H_{min}$, where d^H_{min} is the minimum Hamming distance of $\mathcal{C}$. Hence, a random parsing $\mathcal{P}$ will suffice for achieving full diversity and the transmission scheme reduces to the concatenation of $\mathcal{C}$, a random bit-interleaver $\boldsymbol{\pi}$ of length N, modulator and cyclic mapping of the modulation symbols onto the transmit antennas. However, a STC designed for the fully interleaved channel cannot assure diversity $\delta_\beta N_R$ in a block fading environment, and the less ergodic the channel is, the stronger the potential diversity loss will be [9, 10].

We have therefore proposed robust code design guidelines that make the constructed codes *smart and greedy*, since the code can exploit all the available diversity provided by the transmit and receive antennas as well as the temporal or frequency diversity provided by the channel variations whenever this is present.

BF Channel with Correlated Blocks: OFDM

Now we consider the case of OFDM as a particular case of correlated fading blocks. Prior to designing our coding strategy, we need to know the diversity characteristics in the OFDM channel for a given impulse response. The autocorrelation of the frequency response for a given frequency spacing ϕ is,

$$R_H(\phi) = E\left[H(f)H^*(f-\phi)\right] = \sum_{p=0}^{P-1} E\left[|h(\tau_p)|^2\right] e^{-j2\pi\phi\tau_p},$$

which is the Fourier Transform of the delay intensity profile of the channel $E\left[|h(\tau_p)|^2\right]$, $0 \le p \le P-1$, where τ_p is the delay of the p-th path. Roughly, the frequency diversity is given by W/B_c, where W is the signal bandwidth and B_c is the coherence bandwidth of the channel. However, this is only true in the case of equal power paths and $N_c = P$. Figure 4 shows $R_H(\phi)$ for the BRAN-A channel model [19] and the impulse response of an indoor environment extracted from a ray tracing channel generator [20], both rounded to the nearest symbol-spaced path, with $W = 20$MHz. We see that subcarriers spaced by $W/2$ are almost decorrelated for the BRAN-A channel while for the ray tracing channel they are highly correlated.

[2]Recall that a convolutional code of rate $r = k/n$ may be seen as a *new* convolutional code of rate $r = K/B$ with $K = kB/n$ constructed by simply lumping B/n trellis steps of the original code into one, denoted by hyper-trellis step, and that, in general, the resulting code may have parallel transitions.

[3]In the terminology of [16] for quasistatic channels, this parsing is denoted by natural code/antenna mapping, or natural STC (NSTC).

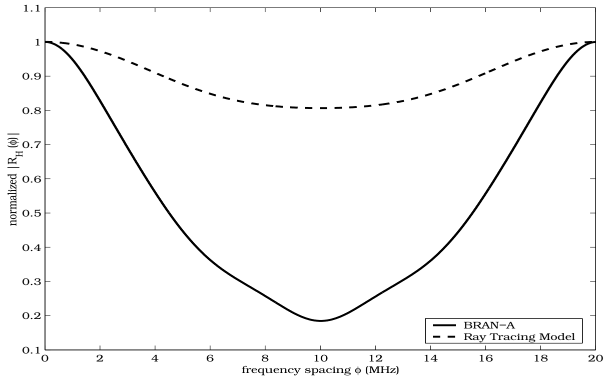

Fig. 4: Normalized autocorrelation of the OFDM channel of BRAN-A and Ray Tracing impulse responses.

Driven by this observations, we will design STCs for OFDM as we proposed in the previous section. We define N_{VB} as the number of (virtual) decorrelated blocks in one OFDM symbol. In order to extract as much diversity as possible from the channel, we will make adjacent symbols in one hyper-trellis step undergo decorrelated fading. Therefore, N_{VB} will be regarded as a design parameter that will allow the code to exploit the additional frequency diversity in the OFDM channel.

Practical Issues

In practice, symbols transmitted at the same instant do interfere. Then, the cyclic interleaver described in the previous section should be designed in the space and block dimensions. We design a cyclic interleaver for which coded symbols are first parsed according to the parsing rule $\mathcal{P}$ and then interleaved. We propose a parsing rule that maps adjacent coded symbols in one hyper-trellis step onto the space (first) and block dimension respectively,

$$(t,\ell)_{\mathcal{P}_1} = \left(|n|_{N_T}, \left| \left\lfloor \frac{n}{N_T} \right\rfloor \right|_{N_B} L_B M + \left\lfloor \frac{n}{N_T N_B} \right\rfloor \right), \qquad (5)$$

for $1 \le n \le N$ (see Figure 5). Modulo-k operation is denoted by $|.|_k$.

time/frequency

antenna

1	9	17	25	33	41	5	13	21	29	37	45
2	10	18	26	34	42	6	14	22	30	38	46
3	11	19	27	35	43	7	15	23	31	39	47
4	12	20	28	36	44	8	16	24	32	40	48

b = 1 b = 2

Fig. 5: Parsing $\mathcal{P}_1$ for $N_T = 4$, $N_B = 2$, $N = 48$ and $M = 1$.

Interleavers should be designed such that for a given coded symbol c of $\mathbf{c} \in \mathcal{C}$, if $c \in \mathcal{B}_{t,b} = \{c : \mathcal{P}(c) = (t,\ell), (b-1)L_B M \le \ell \le bL_B M\}$, then $\boldsymbol{\pi}_t(c) \in \mathcal{B}_{t,b}$. This means that the sub-blocks of $\mathbf{c}$ assigned to different antenna/block pairs are separately interleaved. For the case of independent blocks, $\boldsymbol{\pi}_t$ can be blockwise independent. On the other hand, for correlated fading blocks, interleavers should be designed such that adjacent symbols in a trellis step are *always* assigned to decorrelated fading blocks, i.e., are separated by a decorrelation block spacing. For example, the same blockwise interleaver at each virtual block will assure this condition. In this way, there is no need for a symbol interleaver to further decorrelate the channel [11]. Moreover, in [11], $\mathcal{P}_1$ with $N_B = 1$ is used for the fully interleaved and OFDM channels, as for the quasistatic case. Notice that STCs proposed in [16, 13] for the quasistatic case are particular instances of $\mathcal{P}_1$.

V. Decoding

In the preceding section, we have proposed STC design criteria for the block fading and the OFDM channels assuming a genie aided decoder. Here we propose several iterative decoding strategies and illustrate the corresponding performance-complexity tradeoff. By applying the belief propagation (sum-product) algorithm to the STC dependency graph (in analogy to the CDMA case [12]), the decoder reduces to a maximum-a-posteriori (MAP) soft-input soft-output (SISO) bitwise MIMO demodulator and a MAP SISO decoder of $\mathcal{C}$ that exchange extrinsic information soft messages through the iterations [21]. Bitwise MIMO demodulation computes the extrinsic log-likelihood ratio (LLR_{ext}) of the m-th bit of the constellation symbol transmitted over antenna t at discrete time ℓ as

$$\text{LLR}^{(i)}_{ext}(c_{t,\ell,m}|\mathbf{y}_\ell, \mathbf{H}_b) = \log \frac{\sum_{\mathbf{x}\in\mathcal{X}^t_{m=0}} p(\mathbf{y}_\ell|\mathbf{x}, \mathbf{H}_b) \prod_{t'=1}^{N_T} \prod_{\substack{m'=1\\ m'\neq m}}^{M} \text{P}^{(i-1)}_{ext}(c_{t',\ell,m'})}{\sum_{\mathbf{x}\in\mathcal{X}^t_{m=1}} p(\mathbf{y}_\ell|\mathbf{x}, \mathbf{H}_b) \prod_{t'=1}^{N_T} \prod_{\substack{m'=1\\ m'\neq m}}^{M} \text{P}^{(i-1)}_{ext}(c_{t',\ell,m'})} \quad (6)$$

for $1 \le m \le M, 1 \le t \le N_T, 1 \le \ell \le L$, where $b = \lfloor \frac{\ell}{N_B} \rfloor$, $\mathcal{X}^t_{m=a} = \{\mathbf{x} \in \mathcal{X}^{N_T} \mid \mu^{-1}_m(x_t) = a\}$, $a \in \mathbb{F}_2$, and $\mu^{-1}_m(x_t) = a$ denotes that the m-th position of the bit label of x_t is equal to a, $\text{P}^{(i)}_{ext}(c)$ denotes extrinsic (EXT) probability of the coded symbol c at the i-th iteration with $\text{P}^{(0)}_{ext}(c) = 0.5$. The conditional p.d.f. of the received signal $\mathbf{y}_\ell = \sqrt{\gamma}\mathbf{H}_b\mathbf{x}_\ell + \mathbf{z}_\ell$, given the input signal $\mathbf{x}$ and the b-th block channel $\mathbf{H}_b$ is $p(\mathbf{y}_\ell|\mathbf{x}, \mathbf{H}_b) \propto \exp(-\|\mathbf{y}_\ell - \sqrt{\gamma}\mathbf{H}_b\mathbf{x}\|^2)$. Notice that, $|\mathcal{X}^t_m| = 2^{N_T M-1}$, and therefore, for large N_T and/or large M, the bitwise MIMO demodulation becomes very complex. In practice, the bitwise MIMO demodulation is approximated with the so-called sphere decoder (see [22, 9] and references therein). It reduces the sums in (6) to the *relevant* terms, i.e., the sets $\mathcal{X}^t_m(r) = \{\mathbf{x} \in \mathcal{X}^t_m \mid (\mathbf{x} - \mathbf{x}_c)^H\mathbf{H}^H\mathbf{H}(\mathbf{x} - \mathbf{x}_c) \le r^2\}$, where $\mathbf{x}_c = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y}$ is the center of the sphere, for a given sphere radious r, such that $|\mathcal{X}^t_m(r)| \ll |\mathcal{X}^t_m|$.

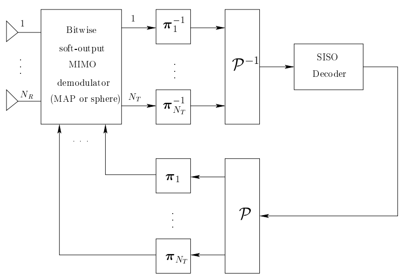

Fig. 6: Iterative decoder with the sum-product algorithm.

In order to further reduce the decoding complexity, we consider IC and linear filtering, for which,

$$\text{LLR}^{(j)}_{ext}(c_{t,\ell,m}|z^{(i)}_{t,b,\ell}, \mathbf{H}_b) = \log \frac{\sum_{x\in\mathcal{X}_{m=0}} p(z^{(i)}_{t,b,\ell}|x, \mathbf{H}_b) \prod_{\substack{m'=1\\ m'\neq m}}^{M} \text{P}^{(j-1)}_{ext}(c_{t,\ell,m'})}{\sum_{x\in\mathcal{X}_{m=1}} p(z^{(i)}_{t,b,\ell}|x, \mathbf{H}_b) \prod_{\substack{m'=1\\ m'\neq m}}^{M} \text{P}^{(j-1)}_{ext}(c_{t,\ell,m'})} \quad (7)$$

for $1 \le m \le M, 1 \le t \le N_T, 1 \le k \le L$, where $\mathcal{X}_{m=a} = \{x \in \mathcal{X} \mid \mu^{-1}_m(x) = a\}$, $z^{(i)}_{t,b,\ell}$ is the output at symbol time ℓ and i-th IC iteration of the front-end linear filter $\mathbf{f}^{(i)}_{t,b}$ of antenna t of channel block b,

$$z^{(i)}_{t,b,\ell} = \mathbf{f}^{(i)\ H}_{t,b}\big(\mathbf{y}_\ell - \sqrt{\gamma}\sum_{t'\neq t}^{N_T} \mathbf{h}_{t',b}\hat{x}^{(i-1)}_{t',\ell}\big) \quad (8)$$

where (dropping antenna and time indexes for simplicity), $\hat{x}^{(i)} = E[x \mid \text{EXT}] = \sum_{x\in\mathcal{X}} x \prod_{m=1}^{M} \text{P}^{(i)}_{ext}(\mu^{-1}_m(x))$ is the minimum mean-square error estimate (conditional mean) of the symbol x given the extrinsic information (briefly denoted by EXT) relative to the bits in the label of x. Iterative BICM decoding for single-input single-output channels is known to provide remarkable performance gain over conventional BICM decoding for some binary labeling rules [7]. Extending this concept to MIMO, we propose a multi-iterative decoder, with IC iterations denoted with index i and demapping iterations denoted by index j (see Figure 7), in a similar way to [23] for turbo-encoded CDMA.

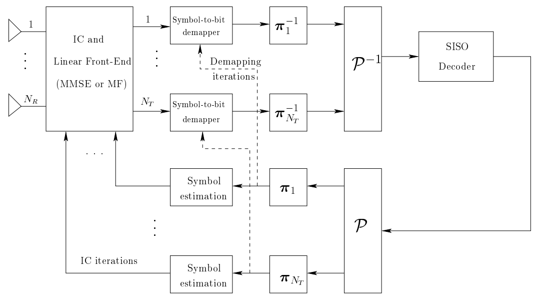

Fig. 7: Iterative decoder with interference cancellation.

Given the analogy with CDMA multiuser detection, we may choose the IC linear front-end $\mathbf{f}^{(i)}_{b,t}$ to be:

1. The *Matched Filter* (MF), $\mathbf{f}^{(i)\ MF}_{t,b} = \mathbf{f}^{MF}_{t,b} = \frac{\sqrt{\gamma}\mathbf{h}_{t,b}}{|\sqrt{\gamma}\mathbf{h}_{t,b}|^2}$, which performs MRC of the receive antennas;
2. The *Unbiased Minimum Mean Squared Error* (MMSE) filter, which minimizes the MSE $E[|x_{t,b} - z_{t,b}|^2]$, is given by $\mathbf{f}^{(i)\ MMSE}_{t,b} = \alpha_{t,b}\sqrt{\gamma}\mathbf{R}^{-1}_b\mathbf{h}_{t,b}$, where $\alpha_{t,b} = (\gamma\mathbf{h}^H_{t,b}\mathbf{R}^{-1}_b\mathbf{h}_{t,b})^{-1}$ is the normalization constant, $\mathbf{R}_b = N_0\mathbf{I} + \sqrt{\gamma}\sum_{t=1}^{N_T}\mathbf{h}_{t,b}\mathbf{h}^H_{t,b}v^{(i-1)}_{t,b}$ is the covariance matrix of the input signal to the filter, and $v^{(i)}_{t,b} = E[|x_{t,b} - \hat{x}^{(i)}_{t,b}|^2]$ is the variance of the residual interference at antenna t and block b at the i-th iteration ($v^{(0)}_{t,b} = 1$) (see [12] and references therein). A practical implementation (and in our simulations) we estimate $v^{(i)}_{t,b}$ as $v^{(i)}_{t,b} \approx 1 - \frac{1}{L_B}\sum_{b=1}^{L_B}|\hat{x}^{(i)}_{t,b}|^2$.

Notice that $\mathbf{f}^{MF}_{t,b}$ does not depend on the iteration index, and thus, it has to be computed once per channel block and transmit antenna. On the contrary, $\mathbf{f}^{MMSE}_{t,b}$ has to be computed once per channel block, transmit antenna and iteration. The proposed algorithm differs from that proposed in [13] in that the latter has to be computed once per symbol interval, transmit antenna and iteration.

VI. Numerical Examples

In this section we show with some numerical examples the performance of BICM STCs for the block fading channel with independent and correlated blocks (OFDM). We run simulations for an input frame length of 128 information bits and we report the FER as a function of E_b/N_0 and SNR$= RE_b/N_0$. The BCJR algorithm is used for SISO decoding of $\mathcal{C}$.

BF Channel with Independent Blocks

Importance of parsing and interleaving Figure 8 shows the FER performance for $N_T = N_R = 2$, $(23, 35)_8$ convolutional code, BPSK modulation, and several values of N_B. We compare the proposed scheme with a random parsing rule that assigns randomly the coded symbols of $\mathbf{c} \in \mathcal{C}$ to the antenna/time array. We also compare our results with separate encoders for each transmit antenna as if each antenna was a separate user in a multiuser CDMA setting. We also plot ML decoding of the NSTC code, which is based on the stacking construction [16] and assigns the output of the t-th generator to the t-th antenna. We observe that the parsing and interleaving rules have a crucial effect on the diversity performance. Indeed, we observe identical performance of the proposed STC with $\mathcal{P}_1$ and MMSE decoding and a NSTC with ML decoding of for the quasistatic channel while there is a substantial loss when a random interleaving (good for a fully interleaved channel) or separate encoders are used. We also report the performance with other values of N_B and $\mathcal{P}_1$, for which $\mathcal{C}$ is SB achieving and we verify that our STCs are smart and greedy, since they exploit the additional diversity whenever there is some.

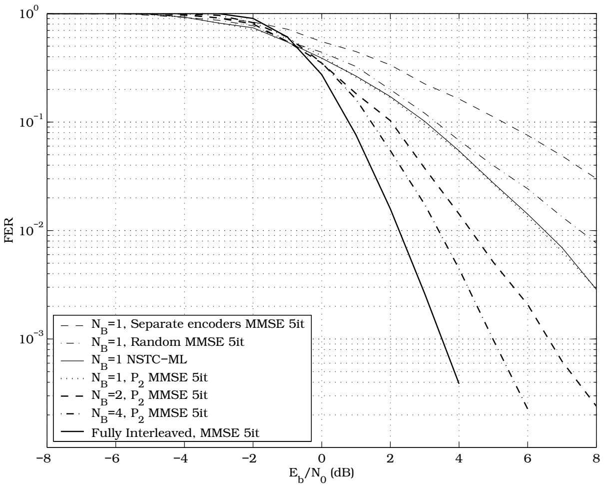

Fig. 8: FER for $N_T = N_R = 2$, $(23, 35)_8$ code, BPSK.

Rank diversity vs. Block diversity Figure 9 shows the FER performance for $N_T = N_R = 4$, $(5, 7, 7, 7)_8$ convolutional code, BPSK modulation for the quasistatic and fully interleaved channel. From the binary rank criterion [16], we know that the resulting NSTC is rank deficient. In [18] we showed that the proposed STC, constructed as row-wise interleaving of the NSTC, is also rank deficient. However, there is a significant difference in the diversity performance. Indeed, for the interleaved code, rank deficient error events do not show their effect in the region of interest, which validates our conjecture on the relevance of the block diversity as design criterion for such codes.

Decoding: Performance vs. Complexity Figure 10 shows the FER performance for $N_T = N_R = 2$, $N_B = 4$, $(23, 35)_8$ convolutional code, $\mathcal{P}_1$, for several modulations, binary labelings and decoders. For the sake of clarity we show these results as function of SNR$(dB) = 10 \log_{10} R + E_b/N_0(dB)$. For the sake of comparison, we also plot the outage probability curves at the corresponding transmission rates.

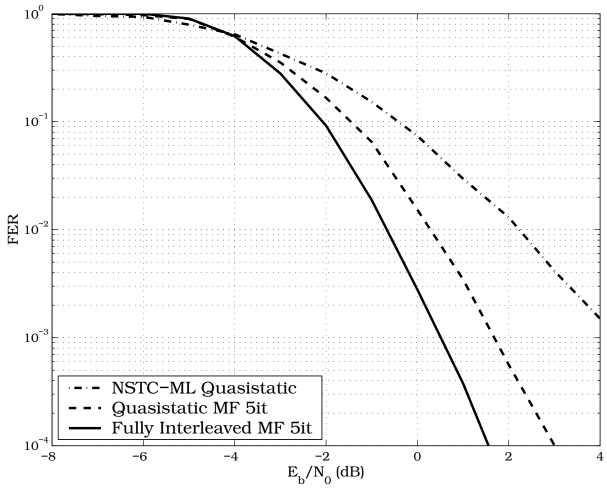

Fig. 9: FER for $N_T = N_R = 4$, $(5, 7, 7, 7)_8$ code and BPSK.

IC and demapping iterations are denoted by it_{IC}, it_{demap} respectively. We first observe that as we increase R, IC receivers with linear filtering, in particular MF, show evident performance degradation, while MAP bitwise demodulation shows always full diversity performance. We have observed that linear receivers for the QPSK case with set partitioning (SP) labeling [7] experience gain with demapping iterations, while in the 16-QAM case, with either Gray (GR) or SP labeling, there was no relevant performance improvement with demapping iterations. Furthermore, GR labeling shows a significant performance advantage over SP labeling for 16-QAM, contradicting the intuitive exportation of the concepts of single-input single-output BICM to the MIMO case. We can also see that the performance curves of the proposed STCs are fairly close to the outage probability only with a 16 states convolutional code. Moreover, we observe that the relative gaps among the different outage probability curves are respected when using practical STC with discrete alphabets. Recalling section III, we see from Figure 2, that $R \approx 2$bit/s/Hz maximizes the throughput at SNR $= 3dB$, with an outage probability of $\approx 7.10^{-2}$, for identical channel conditions and Gaussian inputs. As we can observe, the proposed scheme with QPSK modulation performs very close to this desirable operating point.

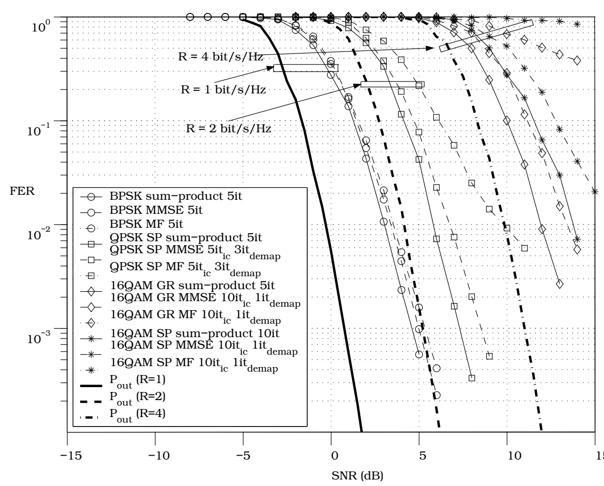

Fig. 10: FER for $N_T = N_R = 2$, $N_B = 4$, $(23, 35)_8$ code, $\mathcal{P}_2$, for several modulations, binary labelings and decoders.

BF Channel with Correlated Blocks: OFDM

Figure 11 illustrates the FER performance for $N_T = N_R = 2$, $(5,7)_8$ convolutional code with BPSK modulation on the BRAN-A and Ray Tracing OFDM channels, with $N_c = 64$, for several values of N_{VB} with the MAP bitwise demodulator. We observe significant diversity gain with $N_{VB} > 1$ virtual blocks per OFDM symbol in both channels with respect to the design in [11], supporting the arguments given in section V. As mentioned before, [11] treats the OFDM channel as quasistatic, since $\mathcal{P}_1$ is used with $N_{VB} = 1$. As we illustrated in Figure 4, there is a relevant diversity difference between the two channel models, and therefore, the relative performance gain with virtual blocks is accordingly related.

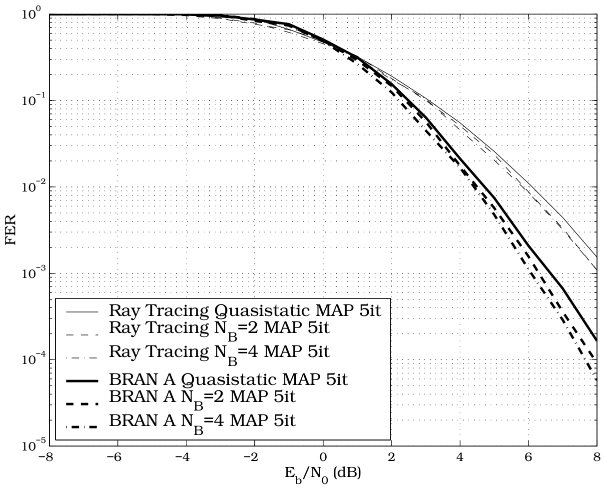

Fig. 11: FER for $N_T = N_R = 2$, $(5,7)_8$ code with BPSK, OFDM BRAN-A and Ray Tracing models, for several N_{VB}.

VII. Conclusions

In this paper we have studied BICM STCs over block fading channels with iterative decoding. BICM STCs are shown to be a simple, pragmatic and flexible approach to construct smart and greedy STCs over block fading channels. We have proposed design criteria based on the algebraic properties of the binary code. We have shown that the proposed code design can be successfully applied to OFDM extracting the maximum amount of diversity from the channel. We have also shown the importance of parser and interleaver design. We have considered belief propagation decoding and IC approximations and we have shown the performance limitations of IC techniques with high transmission rates. This motivates further research on low complexity techniques based on soft-output sphere decoding [22, 9]. We have also shown that binary labelings that perform well in the single-input single-output case, do not show the same performance advantages in the MIMO case.

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