IMPACT OF SIGNAL CONSTELLATION EXPANSION ON ACHIEVABLE DIVERSITY IN QUASISTATIC MULTIPLE ANTENNA CHANNELS

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ABSTRACT

This paper studies the effect of signal constellation expansion on the achievable diversity of space-time codes in quasistatic multiple antenna channels. Signal constellation expansion can be obtained either by increasing the size of the constellation in the complex plane or by using multidimensional linear mappings. By means of two simple space-time code constructions, we provide a comparison of the two options with message passing decoding. We show that multidimensional expansion that inherently achieves full diversity, can be advantageous over complex plane expansion due to its intrinsic design flexibility.

1. INTRODUCTION AND SYSTEM MODEL

Multple antenna transmission has emerged as a key technology to achieve large spectral and power efficiency in wireless communications. In this work, we consider communication in a multiple antenna environment with N_T transmit and N_R receive antennas in quasistatic frequency flat fading. The complex baseband received signal matrix $\mathbf{Y} \in \mathbb{C}^{N_R \times L}$ is given by,

$$\mathbf{Y} = \sqrt{\rho}\mathbf{H}\mathbf{X} + \mathbf{Z}, \tag{1}$$

where L is the blocklength, $\mathbf{X} = [\mathbf{x}_1 \dots \mathbf{x}_{N_T}]^T \in \mathbb{C}^{N_T \times L}$ is the transmitted signal matrix, $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_{N_T}] \in \mathbb{C}^{N_R \times N_T}$, is the fading channel matrix which stays constant during the whole transmission of $\mathbf{X}$ (quasistatic fading), $\mathbf{Z} \in \mathbb{C}^{N_R \times L}$ is a matrix of noise samples i.i.d. $\sim \mathcal{N}_{\mathbb{C}}(0,1)$, and ρ is the average signal to noise ratio (SNR) per transmit antenna. The elements of $\mathbf{H}$ are assumed to be i.i.d. circularly symmetric Gaussian random variables $\sim \mathcal{N}_{\mathbb{C}}(0,1)$ (frequency flat Rayleigh fading). The channel $\mathbf{H}$ is assumed to be perfectly known at the receiver and not known at the transmitter.

The multiple-input multiple-output (MIMO) channel defined by (1) has zero capacity. Consider an ensemble of space-time codes (STC) generated according to the input distribution P_X. We denote the mutual information per channel use (for a fixed $\mathbf{H}$) by $I_{\mathbf{H}}(P_X)$. It can be shown that the minimum achievable error probability for the ensemble in the limit for large block length is given by the *information outage probability* defined as $P_{\rm out}(R) = \Pr(I_{\mathbf{H}}(P_X) \leq R)$ where R is the transmission rate in bits per channel use. When $P_X = \mathcal{N}_{\mathbb{C}}(0,\rho)$ (Gaussian inputs), $I_{\mathbf{H}}(P_X) = \log\det(\mathbf{I} + \rho\mathbf{H}^H\mathbf{H})$. A space-time code $\mathcal{S} \subseteq \mathbb{C}^{N_T \times L}$ is a coding scheme that exploits both temporal and space dimensions in order to achieve reliable communication. The goodness of a STC scheme is usually measured by its ability to approach the outage probability limit.

Conventional code design for quasistatic MIMO channels is based on the ML decoding union bound error probability [1]. The average (over the channel states) pairwise error probability, i.e., the probability of deciding in favor of $\mathbf{X}'$ when $\mathbf{X}$ was transmitted assuming that there are no other codewords, is given by,

$$P(\mathbf{X}' \to \mathbf{X}) \leq G_c \rho^{-d_r N_R} \tag{2}$$

where G_c is the *coding gain* and $d_r N_R$ denotes the diversity gain, where

$$d_r = \min_{\mathbf{X},\mathbf{X}' \in \mathcal{S}} \operatorname{rank}(\mathbf{X} - \mathbf{X}') \tag{3}$$

is the *rank diversity* of the space-time code $\mathcal{S} \subseteq \mathbb{C}^{N_T \times L}$, defined as the set of all possible codewords. Conventional space-time code design searches for *full-diversity* codes $\mathcal{S}$, i.e., $d_r = N_T$, with the largest possible coding gain.

In this work, we study two different approaches to construct full-diversity space-time codes. In particular, we first review a pragmatic construction based on bit-interleaved coded modulation (BICM) [2], which relies on the algebraic properties of the underlying binary code to achieve diversity. Secondly, we consider the concatenation of a coded modulation scheme with an inner code that is linear in the field of complex numbers (linear dispersion (LD) code [3]).In both cases, full-diversity space time codes of any desired spectral efficiency are constructed by suitably expanding the signal constellation. In the first case, we have constellation expansion in the complex plane (Ungerboeck's style expansion), while in the second, we have multidimensional expansion induced by the inner code. By means of message passing decoding, we compare both approaches and we show that, in general, the concatenated LD construction is always advantageous due to its higher design flexibility.

2. PRAGMATIC SPACE-TIME CODES

We consider natural STCs (NSTC) coupled with BICM as a pragmatic way to construct *good* space-time codes (see e.g. [4, 5]). We nickname such scheme BICM NSTC. Such codes are formally defined by a binary block code $\mathcal{C} \subseteq \mathbb{F}_2^N$ of length N and rate r and a spatial modulation function $\mathcal{F} : \mathcal{C} \to \mathcal{S} \subseteq \mathcal{X}^{N_T \times L}$, such that $\mathcal{F}(\mathbf{c}) = \mathbf{X}$, where $\mathcal{X} \subseteq \mathbb{C}$ is the complex signal constellation. We study the case where $\mathcal{F}$ is obtained as the concatenation of a block/antenna parsing function $\mathcal{P} : \mathbb{Z}_+ \to \mathbb{Z}_+^2$ such that $\mathcal{P}(n) = (t, \ell), 1 \leq n \leq N, 1 \leq t \leq N_T, 1 \leq \ell \leq LM$ partitions a codeword $\mathbf{c} \in \mathcal{C}$ into sub-blocks, and blockwise BICM, where each sub-block is independently bit-interleaved and mapped over the signal set $\mathcal{X}$ according to a labeling rule $\mu : \mathbb{F}_2^M \to \mathcal{X}$, such that $\mu(b_1, \dots, b_M) = x$, where $M = \log_2|\mathcal{X}|$ (see Figure 1) [1]. In this case, $N = N_T LM$. The transmission rate (spectral efficiency) of the resulting STC is $R = rN_T M$ bit/s/Hz.

BICM NSTCs are designed assuming a genie aided decoder that produces observables of the transmitted symbols of one antenna, when the symbols from all other antennas are perfectly removed[2]. In this way, the channel decomposes into an equivalent set of N_T single-input single-output non-interfering parallel channels (see [6]

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[1] In the remainder of this paper we shall only consider Gray labeling rules, since they are more efficient (see e.g [4]) in quasistatic channels. This conclusion may be reversed in fully-interleaved channels [5].

[2] The reader will notice the analogy with the case of decision feedback equalization for frequency selective channels, where correct feedback is assumed to design the equalizer filters.

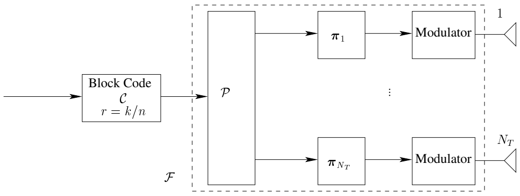

Figure 1: Transmission scheme of BICM NSTC.

and references therein). We define the *block diversity* of a STC $\mathcal{S}$ as the blockwise Hamming distance,

$$\delta_\beta = \min_{\mathbf{X},\mathbf{X}' \in \mathcal{S}} |\{t \in [1,\dots,N_T] \; : \; \mathbf{x}_t - \mathbf{x}'_t \neq 0\}|,$$

i.e., the minimum number of nonzero rows of $\mathbf{X} - \mathbf{X}'$. Then, with a genie aided decoder BICM NSTCs *can* achieve diversity $\delta_\beta N_R$ [6]. Notice that applying BICM within a block, preserves the block diversity of the underlying binary code, since the binary labeling rule μ is a bijective correspondence. Thus, the block diversity of $\mathcal{S}$ is equal to the block diversity of $\mathcal{C}$ provided that the parser $\mathcal{P}$ preserves the block diversity of $\mathcal{C}$. For a binary code $\mathcal{C}$ of rate r over $\mathbb{F}_2$ mapped over B independent blocks, a fundamental upper bound on δ_β is provided by the Singleton bound (SB),

$$\delta_\beta \leq 1 + \lfloor B\,(1-r) \rfloor \tag{4}$$

Consequently, with BICM NSTCs we will search for codes maximizing δ_β, i.e., achieving the SB for all values of B. In quasistatic MIMO channels we let $B = N_T$.

On the other hand, the evaluation of the rank diversity of BICM NSTCs can be a very involved task especially for constellations with $M > 1$. For binary BICM NSTCs, it is however possible to verify through the stacking construction theorem whether a BICM NSTC is full-rank or not. In particular consider the following.

Stacking construction [7]. Let $\mathbf{G}_1,\dots,\mathbf{G}_{N_T}$ be binary matrices $\in \mathbb{F}_2^{K\times N}$, and consider the binary linear code of rate $K/(N_T N)$ generated by $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_{N_T}]$. Let the code words $\mathbf{c} = \mathbf{b}\mathbf{G}$ of $\mathcal{C}$, where $\mathbf{b} \in \mathbb{F}_2^K$, be parsed as

$$\mathbf{C} = \mathcal{F}(\mathbf{c}) = \begin{bmatrix} \mathbf{b}\mathbf{G}_1 \\ \mathbf{b}\mathbf{G}_2 \\ \vdots \\ \mathbf{b}\mathbf{G}_{N_T} \end{bmatrix}$$

Then, if for all $a_1,\dots,a_{N_T} \in \mathbb{F}_2$ non all zero, the $K \times N$ matrix

$$\mathbf{M} = \bigoplus_{t=1}^{N_T} a_t \mathbf{G}_t$$

($\bigoplus$ indicates addition in the binary field $\mathbb{F}_2$) has rank K, then the binary BICM NSTC obtained from $\mathcal{C}$ with the above parsing has full rank-diversity $d_r = N_T$ (the condition is necessary and sufficient).

Now let $\mathbf{G}_t^{\mathcal{C}}$ be the t-th generator matrix of $\mathcal{C}$. The generator matrices $\mathbf{G}'_t$ of BICM NSTCs can be easily obtained from $\mathbf{G}_t^{\mathcal{C}}$, by simply applying the permutation $\boldsymbol{\pi}_t$ to the columns of $\mathbf{G}_t^{\mathcal{C}}$, for $t = 1,\dots,N_T$. We can now apply the stacking construction theorem with the generator matrices of the BICM NSTC $\mathbf{G}'_t$, in order to check for its a priori diversity performance under ML decoding. Notice that for BICM NSTCs we have that $d_r \leq \delta_\beta \leq N_T$.

3. LD CONCATENATED CODES

In this section we consider the case where the codewords $\mathbf{X}$ of the STC $\mathcal{S}$ are obtained from the concatenation of an outer coded modulation scheme $\mathcal{C}^O \subseteq \mathcal{X}^n$ of rate r_O and length n with an inner LD code. The inner code is formed by a parser $\mathcal{P}$, that partitions the codewords $\mathbf{c} \in \mathcal{C}^O$ into sub-blocks $\mathbf{c}[j] = [c_1[j],\dots,c_Q[j]]$, $j = 1,\dots,J$ of length Q, with $K = n/Q$ and by a LD space-time modulation function $\mathcal{F}$ defined by,

$$\mathbf{S}[j] = \mathcal{F}(\mathbf{c}[j]) = \sum_{q=1}^{Q} (c_q[j]\mathbf{G}_q), \tag{5}$$

where $\mathbf{G}_q \in \mathbb{C}^{N_T \times T}$ are the LD code generator matrices. Finally, the overall space-time codeword is given by $\mathbf{X} = [\mathbf{S}[1] \dots \mathbf{S}[J]]$. Then,

$$\mathbf{Y}[j] = \mathbf{H}\mathbf{S}[j] + \mathbf{Z}[j], \; j = 1,\dots,J. \tag{6}$$

Equation (1) can be rewritten as a virtual MIMO channel with Q inputs and $N_R^v = N_R T$ outputs as,

$$\mathbf{y}[j] = \mathcal{H}\mathbf{c}[j] + \mathbf{z}[j], \;\; j = 1,\dots,J \tag{7}$$

where $\mathcal{H} \in \mathbb{C}^{N_R^v \times Q}$ is the equivalent channel matrix given by,

$$\mathcal{H} = [I_T \otimes \mathbf{H}]\mathbf{G} \tag{8}$$

where $\otimes$ is the Kronecker product, $\mathbf{G} \in \mathbb{C}^{N_T T \times Q}$ is the suitably reformatted generator matrix of the LD code, $\mathbf{y}[k] = \mathrm{vec}(\mathbf{Y}[k])$, $\mathbf{z}[k] = \mathrm{vec}(\mathbf{Z}[k])$, $N_R^v = N_R T$ is the number of virtual receive antennas, and $\mathrm{vec}(\mathbf{A}) = [\mathbf{a}_1^T \dots \mathbf{a}_l^T]^T$, for a matrix $\mathbf{A} = [\mathbf{a}_1 \dots \mathbf{a}_l]$. Will refer to Q as the number of virtual transmit antennas.

In this work, we consider that $\mathcal{C}^O$ is obtained by BICM, i.e, a binary code $\mathcal{C} \in \mathbb{F}_2^N$ of rate r whose bit-interleaved codewords are mapped onto the signal set $\mathcal{X}$ according to the binary labeling rule $\mu : \mathbb{F}_2^M \to \mathcal{X}$ [2]. As inner LD code, we will use the threaded algebraic space-time (TAST) constellations of [8]. We nickname such a transmission scheme as BICM TAST (see Figure 2).

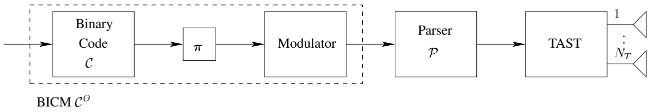

Figure 2: Transmission scheme of BICM TAST.

The aforementioned algebraic STCs rely on the threaded layering first introduced in [9] for threaded STCs (TSTC). In TSTC, a number of component encoders (or layers) N_L and the component codewords $\mathbf{c}_\ell \in \mathcal{C}_\ell$, $\ell = 1,\dots,N_L$ are mapped onto the array $\mathcal{T}_{N_T,N_L,N_L}$ following the (layer, antenna, time) indexing triplet

$$\mathcal{P}(\ell,n) = (\ell, |t+\ell-1|_{N_T}, t), \; 1 \leq t \leq T, \; \ell = 1,\dots,N_L,$$

thus having full spatial and temporal spans. Modulo-k operation is denoted by $|.|_k$. The encoding functions corresponding to $\mathcal{C}_\ell$, $\ell = 1,\dots,N_L$ are $\gamma_\ell = \phi_\ell \mathbf{M}$, where $\mathbf{M}$ is a rate one full-diversity linear algebraic rotation and ϕ_ℓ, $\ell = 1,\dots,N_L$ scalar complex coefficients chosen to be Diophantine numbers that ensure that TAST codes achieve full diversity with maximum-likelihood (ML) decoding (see [8] for details). In TAST codes, $N_L = \min(N_T, N_R)$, $T = N_T$, $Q = N_L N_T$, and ϕ_ℓ, $\ell = 1,\dots,N_L$ are chosen such that $|\phi_\ell| = 1$, $\ell = 1,\dots,N_L$. The transmission rate of the resulting STC is $R = rN_L M$ bit/s/Hz. Notice that following this *divide and conquer* design, full diversity is always guaranteed by the inner code, while coding gain is left to the outer coded modulation.

4. MESSAGE PASSING DECODING

Because of the pseudo-random bit interleaver present in both schemes, ML decoding of BICM NSTC or BICM TAST is of unaffordable complexity. We therefore resort to iterative techniques based on a factor graph representation. In analogy to the case of multiuser receivers for CDMA [10], applying the belief propagation (BP) algorithm to the STC dependency graph, yields several receivers that approximate the optimal maximum a posteriori (MAP) detection rule. In particular, exact BP reduces the overall receiver to a MAP soft-input soft-output (SISO) bitwise demodulator and a MAP SISO decoder of $\mathcal{C}$, that exchange extrinsic information probability messages through the iterations. When $\mathcal{C}$ is a trellis code, the messages are efficiently computed by the forward-backward algorithm (BCJR) with linear complexity in N [11]. For example, in the case of BICM NSTC[3] the log-likelihood ratio (LLR) message produced at the i-th iteration by the MAP SISO bitwise demodulator for

[3]The case of BICM TAST is completely analogous, and it suffices to replace N_T by Q, L by J and $\mathbf{H}$ by $\mathcal{H}$.

the decoder of $\mathcal{C}$, corresponding to the m-th bit of the constellation symbol transmitted over antenna t at discrete time ℓ is given by,

$$\mathrm{LLR}_{ext}^{(i)}(c_{t,\ell,m}|\mathbf{y}_\ell,\mathbf{H}) = \log \frac{\sum_{\mathbf{x}\in\mathcal{X}^t_{m=0}} p(\mathbf{y}_\ell|\mathbf{x},\mathbf{H}) \prod_{t'=1}^{N_T}\prod_{\substack{m'=1\\ m'\neq m \text{ if } t=t'}}^{M} \mathrm{P}_{ext}^{(i-1)}(c_{t',\ell,m'})}{\sum_{\mathbf{x}\in\mathcal{X}^t_{m=1}} p(\mathbf{y}_\ell|\mathbf{x},\mathbf{H}) \prod_{t'=1}^{N_T}\prod_{\substack{m'=1\\ m'\neq m \text{ if } t=t'}}^{M} \mathrm{P}_{ext}^{(i-1)}(c_{t',\ell,m'})}$$

for $1 \leq m \leq M, 1 \leq t \leq N_T, 1 \leq \ell \leq L$, where $\mathcal{X}^t_{m=a}$ is the set of N_T-dimensional symbols for which the m-th bit of the symbol transmitted over antenna t equal to a, $\mathrm{P}_{ext}^{(i)}(c)$ denotes extrinsic (EXT) probability (provided by the SISO decoder of $\mathcal{C}$) of the coded binary symbol c at the i-th iteration with $\mathrm{P}_{ext}^{(0)}(c) = 0.5$. The conditional p.d.f. of the received signal $\mathbf{y}_\ell = \sqrt{\rho}\mathbf{H}\mathbf{x}_\ell + \mathbf{z}_\ell$, given the input signal $\mathbf{x}$ and the channel $\mathbf{H}$ is $p(\mathbf{y}_\ell|\mathbf{x},\mathbf{H}) \propto \exp(-\|\mathbf{y}_\ell - \sqrt{\rho}\mathbf{H}\mathbf{x}\|^2)$. Exact BP is of exponential complexity in M and N_T for BICM NSTC and Q for BICM TAST and it is usually approximated by soft-output sphere decoding techniques (see e.g. [12] for recent results on the subject).

In this work we also consider lower-complexity algorithms based on iterative interference cancellation (IC) and linear filtering. In this case, the LLR message to the decoder of $\mathcal{C}$ is given by,

$$\mathrm{LLR}_{ext}^{(i)}(c_{t,\ell,m}|z_{t,\ell}^{(i)},\mathbf{H}) = \log \frac{\sum_{x\in\mathcal{X}_{m=0}} p(z_{t,\ell}^{(i)}|x,\mathbf{H}) \prod_{\substack{m'=1\\ m'\neq m}}^{M} \mathrm{P}_{ext}^{(i-1)}(c_{t,\ell,m'})}{\sum_{x\in\mathcal{X}_{m=1}} p(z_{t,\ell}^{(i)}|x,\mathbf{H}) \prod_{\substack{m'=1\\ m'\neq m}}^{M} \mathrm{P}_{ext}^{(i-1)}(c_{t,\ell,m'})}$$

for $1 \leq m \leq M, 1 \leq t \leq N_T, 1 \leq k \leq L$, where now $\mathcal{X}_{m=a}$ is the set of all constellation points of $\mathcal{X}$ with the m-th bit of the label equal to a, $z_{t,\ell}^{(i)}$ is the output at symbol time ℓ and i-th iteration of the front-end linear filter $\mathbf{f}_t^{(i)}$ of antenna t after IC,

$$z_{t,\ell}^{(i)} = \mathbf{f}_t^{(i)\,H}\big(\mathbf{y}_\ell - \sqrt{\rho}\sum_{t'\neq t}^{N_T} \mathbf{h}_{t'}\hat{x}_{t',\ell}^{(i-1)}\big),$$

where (dropping antenna and time indexes for simplicity),

$$\hat{x}^{(i)} = E[x\,|\,\mathrm{EXT}] = \sum_{x\in\mathcal{X}} x \prod_{m=1}^{M} \mathrm{P}_{ext}^{(i)}(c_m)$$

is the minimum mean-square error estimate (conditional mean) of the symbol x given the extrinsic information (briefly denoted by EXT) relative to the bits in the label of x.

In particular, we consider minimum mean squared error (MMSE) IC, for which the filter at the i-th iteration corresponding to the t-th antenna is given by,

$$\mathbf{f}_t^{(i)} = \alpha_t\sqrt{\rho}\mathbf{R}^{-1}\mathbf{h}_t, \tag{9}$$

where $\alpha_t = (\rho\mathbf{h}_t^H\mathbf{R}^{-1}\mathbf{h}_t)^{-1}$ is the normalization constant, $\mathbf{R} = \mathbf{I} + \sqrt{\rho}\sum_{t=1}^{N_T}\mathbf{h}_t\mathbf{h}_t^H v_t$ is the covariance matrix of the input signal to the filter, and $v_t = E[|x_t - \hat{x}_t|^2]$ is the variance of the residual interference at virtual antenna t (see [10] and references therein). A practical implementation (and in our simulations) we estimate v_t as $v_t \approx 1 - \frac{1}{L}\sum_{\ell=1}^{L}|\hat{x}_t[\ell]|^2$. Notice that $\mathbf{f}_t^{(i)}$ has to be computed once per virtual transmit antenna and iteration. The proposed algorithm differs from that proposed in [9] in that the latter has to be computed once per symbol interval, transmit antenna and iteration.

5. EXAMPLES

In this section we provide several numerical examples obtained by computer simulation that illustrate the effect of the constellation expansion on the achievable diversity of BICM NSTC and BICM TAST. For the sake of comparison, we include the outage probability curves with Gaussian inputs at the corresponding spectral efficiency. Unless otherwise specified we take frames of 128 information bits and 5 decoding iterations.

Figure 3 reports the frame error rate (FER) as a function of E_b/N_0 in a MIMO channel with $N_T = 4$ and $N_R = 4$, using BPSK moduilation and the 4 states $(5,7,7,7)_8$ convolutional code of $r = 1/4$. The overall spectral efficiency is $R = 1$bit/s/Hz. Clearly, the block diversity of $\mathcal{C}$ is $\delta_\beta = 4$. In dased-dotted line we show the FER for the NSTC with ML decoding. Recall that the NSTC array is constructed using identity permutations [7], and therefore ML decoding is possible using the Viterbi algorithm. Applying the stacking construction theorem yields that the NSTC code is rank defficient. We have also applied the theorem to BICM NSTC with a large number of randomly generated interleaver permutations, and none of them gave a full-rank code. However, as the curves in the figure show, in the FER region of interest full-diversity performance is achieved with two suboptimal iterative receivers (BP and MMSE-IC). This simple example shows the key role that the block diversity plays to achieve full diversity in BICM NSTC.

Figure 4 shows the FER performance in a $N_T = 2$ and $N_R = 2$ MIMO channel with the 4 states $(5,7)_8$ convolutional code of $r = 1/2$ with QPSK and 16-QAM modulations with Gray mapping. The spectral efficiencies are $R = 2, 4$ bit/s/Hz respectively. The block diversity of $\mathcal{C}$ is $\delta_\beta = N_T = 2$, and therefore, BICM NSTC should achieve full diversity with a *good* decoder. On the other hand, the diversity of BICM TAST is given by the TAST constellation and the coded modulation is only responsible for an horizontal shift of the error curve, i.e, the coding gain. Notice that, in such concatenated scheme, we can set $\phi_\ell = 1,\ \ell = 1,\dots,N_L$ without any noticeable difference in performance, since the outer code removes most of the rank-deficient error events of the inner code. In this way, it suffices to find a good rotation matrix $\mathbf{M}$ in orfer to construct good BICM TAST codes. As we observe, under BP decoding and the same configuration, since BICM NSTC has full block diversity, all schemes perform almost identical regardless of their different nature. Notice that LD constellations induce an increased peak-to-average power ratio, which can make them impractical for applications where the power amplifier is operated close to saturation.

In Figure 5 we report the FER performance in a $N_T = 2$ and $N_R = 2$ MIMO channel with the 4 states convolutional codes and QPSK and 8-PSK modulations with Gray mapping for an overall spectral efficiency of $R = 3$bit/s/Hz. In this case the frame is taken to be 132 information bits long. This figure clearly illustrates the effect of constellation expansion to achieve full diversity. In fact, in order to achieve $R = 3$bit/s/Hz with QPSK, we need the rate of $\mathcal{C}$ be $r = 3/4$. As we observe, under such configuration BICM TAST achieves full diversity due to its inherent multidimensional constellation expansion. On the other hand, the diversity of BICM NSTC is governed by the Singleton bound (which in this case yields $\delta_\beta = 1$) and therefore under this configuration it does not achieve full-diversity. However, $R = 3$bit/s/Hz can also be achieved by using a rate $r = 1/2$ code (which has $\delta_\beta = N_T = 2$) and expanding the signal constellation in the complex plane, i.e., using 8-PSK modulation. As we observe, in this case, BICM NSTC achieves full diversity. However, it pays about a 1dB penalty in average power for the expansion with respect to the BICM TAST.

Figure 6 shows the FER performance of BICM NSTC and BICM TAST in a MIMO channel with $N_T = 4$ and $N_R = 4$, with the 4 states $(5,7,7,7)_8$ convolutional code of $r = 1/4$, using 16 and 64 QAM modulations with Gray mapping and MMSE-IC decoding. The corresponding spectral efficiencies are $R = 4, 6$ bit/s/Hz. In the case of 64-QAM we have considered frames of 120 information bits. We also plot the simulated matched filter bound (MFB), i.e., an ideal genie aided decoder. In this example we observe that a new effect arises, namely, for too large spectral efficiency, even if the transmission schemes ensure full diversity, the MMSE-IC decoder is not able to remove the interference and achieve the correct slope. The characterization of the thresholds of the spectral efficiency for which the MMSE-IC is able to perform close to ML or BP is a very diffiult problem and at present there is no satisfactory explanation. In [13] we have derived a semi-analytical method based on density evolution and bounding techinques, which however is as complex as simulation due to the outer expectation over the quasistatic fading.

6. CONCLUSIONS

In this paper we have illustrated the effect of signal constellation expansion on the achievable diversity in quasistatic MIMO channels. In particular we have compared complex plane expansion and lattice-based expansion. We have shown that under message passing decoding, concatenated STCs with inner LD codes benefit from a higher design flexibility and show some performance advantage. However, in the same setting, both schemes perform equivalent. Thus, since LD-based methods induce an increased peak-to-average power ratio, we shall prefer the first approach in applications where the power amplifier is driven close to saturation.

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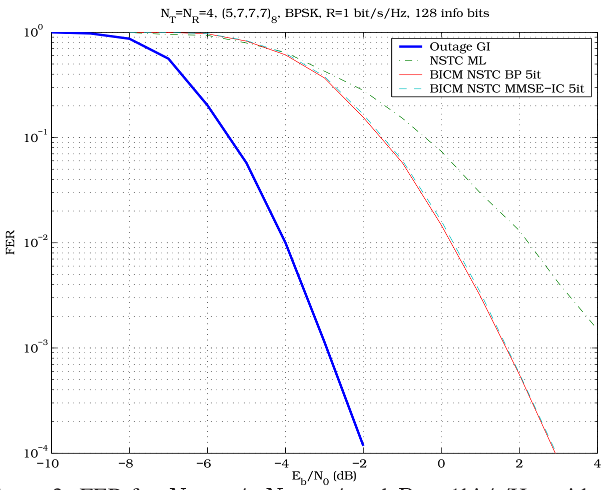

Figure 3: FER for $N_T = 4$, $N_R = 4$ and $R = 1$bit/s/Hz, with the $(5, 7, 7, 7)_8$ convolutional code and BPSK modulation.

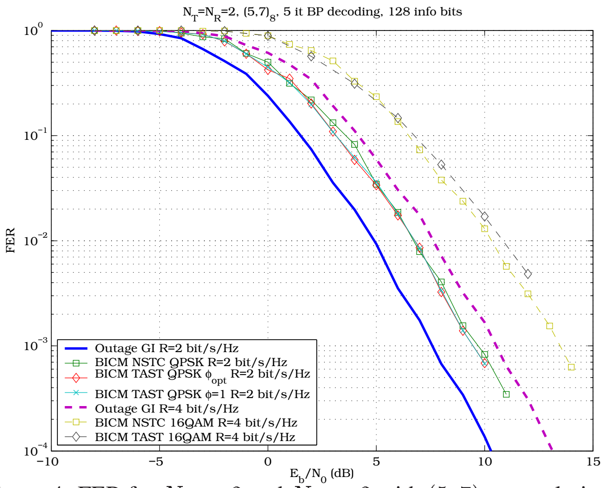

Figure 4: FER for $N_T = 2$ and $N_R = 2$ with $(5, 7)_8$ convolutional code, QPSK and 16-QAM with Gray mapping.

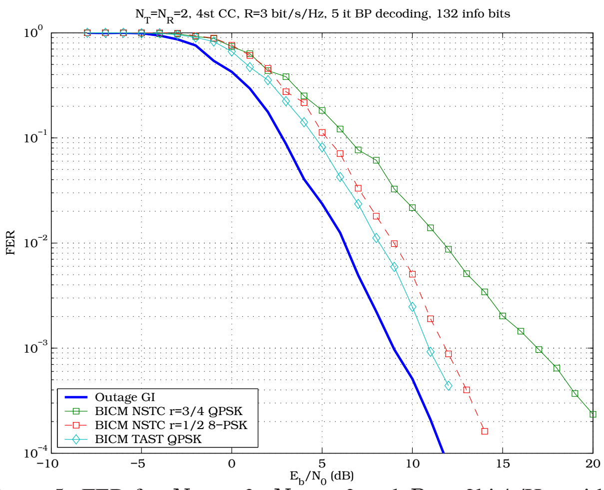

Figure 5: FER for $N_T = 2$, $N_R = 2$ and $R = 3$bit/s/Hz, with 4 states convolutional codes, QPSK and 8-PSK with Gray mapping.

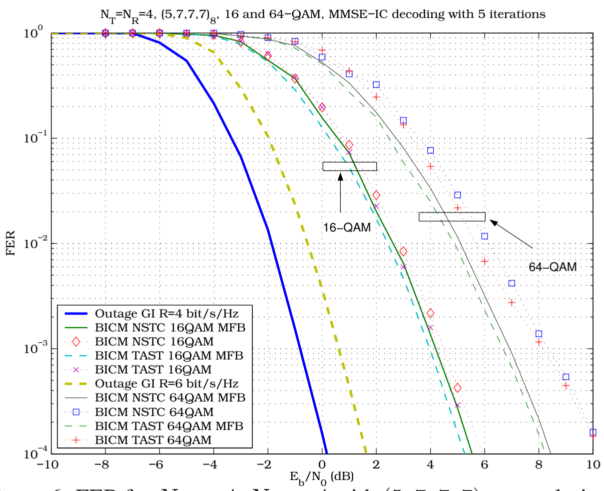

Figure 6: FER for $N_T = 4$, $N_R = 4$ with $(5, 7, 7, 7)_8$ convolutional code, 16-QAM and 64-QAM with Gray mapping, with MMSE-IC decoding.