Turbo-like Codes for the Block-Fading Channel

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We consider a single-input single-output block-fading channel with N_B fading blocks [1]. The received signal can be compactly expressed in the matrix form

$$\mathbf{Y} = \sqrt{\rho}\,\mathbf{H}\mathbf{X} + \mathbf{Z} \tag{1}$$

where $\mathbf{Y} \in \mathbb{C}^{N_B \times L}$, $\mathbf{X} \in \mathbb{C}^{N_B \times L}$, $\mathbf{H} = \mathrm{diag}(h_1, \ldots, h_{N_B}) \in \mathbb{C}^{N_B \times N_B}$ and $\mathbf{Z} \in \mathbb{C}^{N_B \times L}$. The b-th block fading coefficient is denoted by h_b and the noise $\mathbf{z}_b$ is i.i.d. proper complex Gaussian, with components $\sim \mathcal{N}_{\mathbb{C}}(0,1)$. We assume normalized fading, such that $\mathbb{E}[|h_b|^2] = 1$. Therefore, the *average* signal-to-noise ratio (SNR) is ρ and the *instantaneous* SNR on block b is given by $|h_b|^2\rho$. The collection of all possible transmitted codewords $\mathbf{X}$ forms a coded modulation scheme over $\mathcal{X}$. We study schemes $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ obtained by concatenating a binary linear code $\mathcal{C}$ of length $N_B L M$ and rate r bit/symbol with a memoryless one-to-one symbol mapper $\mu : \mathbb{F}_2^M \to \mathcal{X}$, with $M = \log_2 |\mathcal{X}|$. The resulting coding rate (in bit/complex dimension) is given by $R = rM$.

We define the SNR reliability function $d_B^\star$ as the maximum achievable SNR exponent of error probability for codes in a given family of interest [3]. Namely, we define

$$d_B^\star \triangleq \sup_{\mathcal{C} \in \mathcal{F}} \lim_{\rho \to \infty} \frac{-\log P_e(\rho, \mathcal{C})}{\log \rho} \tag{2}$$

where $P_e(\rho, \mathcal{C})$ is the error probability of a given coding scheme $\mathcal{C}$, and the supremum is taken over all coding schemes in the family $\mathcal{F}$. For discrete signal sets and for bit-interleaved coded modulation (BICM) [2] we have the following results:

Theorem 1 *Consider the block-fading channel (1) with i.i.d. Rayleigh fading and input signal set $\mathcal{X}$ of cardinality 2^M. The SNR reliability function of the channel is upperbounded by the Singleton bound*

$$d_B^\star(R) \le \delta_B(R) \triangleq 1 + \left\lfloor N_B \left(1 - \frac{R}{M}\right) \right\rfloor \tag{3}$$

The random coding SNR exponent of the coded modulation ensemble $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ defined previously, with block length $L(\rho)$ satisfying $\lim_{\rho \to \infty} \frac{L(\rho)}{\log \rho} = \beta$ and rate R, is lowerbounded by $\beta N_B M \log(2)\left(1 - \frac{R}{M}\right)$ when $0 \le \beta < \frac{1}{M \log(2)}$ and by $\delta_B(R) - 1 + \min\left\{1, \beta M \log(2)\left[N_B\left(1 - \frac{R}{M}\right) - \delta_B(R) + 1\right]\right\}$ when $\frac{1}{M\log(2)} \le \beta < \infty$. Furthermore, the SNR random coding exponent of the associated BICM channel satisfies the same lower bounds.

Corollary 1 *The SNR reliability function of the block-fading channel with input $\mathcal{X}$ and of the associated BICM channel is given by $d_B^\star(R) = \delta_B(R)$ for all $R \in (0, M]$, except for the N_B discontinuity points of $\delta_B(R)$, i.e., for the values of R for which $N_B(1 - R/M)$ is an integer.*

Fig. 1 shows $\delta_B(R)$ (Singleton bound) and the random coding lower bounds for $\beta M \log(2) = 1/2$ and $\beta M \log(2) = 2$, in the case $N_B = 8$ and $M = 4$. It can be observed that as β increases, the random coding lower bound coincides over a larger and larger support with the Singleton upper bound. However, in the discontinuity points it will never coincide.

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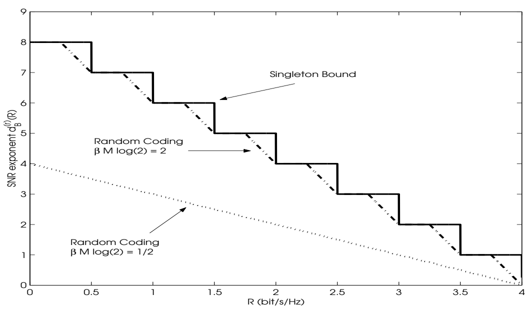

Fig. 1: SNR reliability function and random coding exponents.

We say that a code ensemble $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ is good if for $L \to \infty$ its error probability converges to the outage probability, while if it shows a fixed SNR gap (independent of L) we say that $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ is weakly good. In [4] we provide a sufficient condition for weak goodness based on asymptotic multivariate weight enumerators.

We consider the coded modulation family $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ of blockwise concatenated codes (BCC) (see Fig. 2). The binary linear outer code $\mathcal{C}^O \in \mathbb{F}_2^{L_O}$ of rate r_O is partitioned into N_B blocks of length L_O/N_B. The blocks are separately interleaved and fed to N_B binary inner encoders $\mathcal{C}^I \in \mathbb{F}_2^{L_I}$ of rate r_I. Finally, the output of each component inner code is mapped onto a sequence of signal components in $\mathcal{X}$ by the modulator mapping μ. The proposed BCCs significantly outperform conventional serial and parallel turbo codes in the block-fading channel. Differently from the AWGN and fully-interleaved fading cases, iterative decoding performs very close to ML on the block-fading channel, even for relatively short block lengths. Moreover, at constant decoding complexity per information bit, BCCs are shown to be weakly good, while standard block codes obtained by trellis-termination of convolutional codes have a gap from outage that increases with the block length: this is a different and more subtle manifestation of the so-called "interleaving gain" of turbo-like codes.

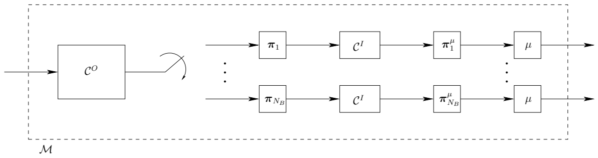

Fig. 2: Code structure for BCCs.

References

[1] L. H. Ozarow, S. Shamai and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. on Vehicular Tech.*, vol. 43, no. 2, pp. 359–378, May 1994.

[2] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. on Inform. Theory*, vol. 44, no. 3, pp. 927–946, May 1998.

[3] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 57, pp. 1073–1096, May 2003.

[4] A. Guillén i Fàbregas and G. Caire, "Coded Modulation in the Block-Fading Channel: Coding Theorems and Explicit Constructions," *submitted to IEEE Trans. on Inform. Theory*, 2004.