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Turbo-like Codes are Good for the Block-Fading Channel

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Abstract

This paper studies turbo-like coded modulation in the block-fading channel. In particular we show that the performance of such coded modulation schemes is close to the information outage probability of the channel for any block length, as opposed to standard coded modulation schemes based on trellis-terminated convolutional codes. For large block length, asymptotic performance of the proposed structure is given by the distribution of its decoding threshold. By using asymptotic weight enumerator techniques, we derive the asymptotic maximum-likelihood performance of the proposed codes.

1. INTRODUCTION

The block-fading channel was introduced in [1] (see also [2]) in order to model slowly-varying fading, where codewords span only a fixed number N_B of fading degrees of freedom, irrespectively of the code block length. This model is particularly relevant in wireless communications situations involving slow time-frequency hopping (e.g., GSM, EDGE) or multicarrier modulation using orthogonal frequency division multiplexing (OFDM). More in general, despite its extreme simplification, it serves as a useful model to develop coding design criteria which turn out to be useful in more general settings of correlated slowly-varying fading.

Strictly speaking, the block-fading channel has zero capacity, since, there is an irreducible probability that the transmitted data rate is not supported by the channel, namely the *information outage probability*. Therefore, for large block length, the probability of error will be at least as large as the outage probability. Therefore, the goodness of coded modulation schemes over the block-fading channel is measured by their ability to approach the outage probability limit for large block length.

We study a family of codes based on the blockwise concatenation of two component encoders through interleavers. Such codes, are known to achieve the optimal rate-diversity tradeoff given by the Singleton bound [3]. In this paper we show that, for such turbo-like structure, by using asymptotic weight enumerator techniques, the word-error rate (WER) of the proposed codes is almost independent of the block length, while the component encoders are fixed, i.e., the decoding complexity of the BP decoder is linear with the block length. On the contrary, in the case of block codes obtained by trellis termination of trellis codes, the WER increases (roughly linearly) with the block length for linear decoding complexity. We interpret this fact as another manifestation of the so-called "interleaving gain" typical of turbo codes, even though, in block-fading, no "waterfall" behavior of the error curve is visible, even for very large block length.

2. SYSTEM MODEL AND OPTIMAL RATE-DIVERSITY TRADEOFF

We consider a single-input single-output block-fading AWGN channel model [1] with N_B fading blocks, where each block has length L complex dimensions. Fading is flat, constant on each block, and i.i.d. on different blocks. The discrete-time complex baseband equivalent channel model is given by

$$\mathbf{y}_b = \sqrt{\rho}\, h_b\, \mathbf{x}_b + \mathbf{z}_b \ \ , \ b = 1, \dots, N_B \qquad (1)$$

where $\mathbf{y}_b, \mathbf{x}_b, \mathbf{z}_b \in \mathbb{C}^L$ denote the b-th received, transmitted and noise vectors respectively, h_b denotes the b-th block fading coefficient and the noise components are i.i.d. complex circularly-symmetric Gaussian $\sim \mathcal{N}_{\mathbb{C}}(0,1)$. We consider Rayleigh fading, for which $h_b \sim \mathcal{N}_{\mathbb{C}}(0,1)$.

We consider codes constructed over a complex signal-set $\mathcal{X}$ (e.g., QAM/PSK) of cardinality 2^M, i.e.,

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the components of the vectors $\mathbf{x}_b$ are points in the constellation $\mathcal{X}$. The overall codeword block length is $N_B L$ (complex dimensions). Therefore, each codeword spans at most N_B independent fading coefficients. Without loss of generality, we assume normalized fading, such that $\mathbb{E}[|h_b|^2] = 1$ and unit-energy signal set $\mathcal{X}$ (i.e., $2^{-M}\sum_{x\in\mathcal{X}}|x|^2 = 1$). Therefore, ρ denotes the *average* received SNR and the *instantaneous* SNR on block b is given by $\gamma_b\rho$, where $\gamma_b \triangleq |h_b|^2$ denotes the fading *power gain*.

It can be shown that the minimum achievable error probability is given by the *information outage probability* defined as $P_{\text{out}}(\rho, R) = \Pr(I_{\mathbf{h}}(P_X) \le R)$, where R is the transmission rate in bits per channel use and $I_{\mathbf{h}}(P_X)$ is the *instantaneous* mutual information for a given input distribution P_X [1]. The goodness of a coding scheme is then measured by the SNR gap from the outage probability for large block length L. In particular, we say that a coded modulation scheme is *good* for the block-fading channel if for $L \to \infty$ the word error probability (WER) shows a fixed gap from $P_{\text{out}}(\rho, R)$ asymptotically independent of L.

For coded modulation schemes over discrete constellations, the optimal rate-diversity tradeoff is given by the SNR reliability function (optimal diversity)

$$d_B^\star \triangleq \sup_{\mathcal{C}\in\mathcal{F}} \lim_{\rho\to\infty} \frac{-\log P_e(\rho,\mathcal{C})}{\log\rho}$$

for a coded modulation family $\mathcal{F}$, and can be stated as follows [3, 4],

Theorem 1 *Consider the block-fading channel (1) with i.i.d. Rayleigh fading and input signal set $\mathcal{X}$ of cardinality 2^M. The SNR reliability function of the channel is upperbounded by the Singleton bound*

$$d_B^\star(R) \le d_{SB}(R) \triangleq 1 + \left\lfloor N_B\left(1 - \frac{R}{M}\right)\right\rfloor \quad (2)$$

The random coding SNR exponent $d_B^{(r)}(R)$ of the coded modulation ensemble $\mathcal{M}(\mathcal{C},\mu,\mathcal{X})$ defined previously, with block length $L(\rho)$ satisfying $\lim_{\rho\to\infty}\frac{L(\rho)}{\log\rho} = \beta$ and rate R, is lowerbounded by

$$\beta N_B M \log(2)\left(1 - \frac{R}{M}\right),$$

for $0 \le \beta < \frac{1}{M\log(2)}$ and

$$d_{SB}(R) - 1 + \min\left\{1, \beta M\log(2)\left[N_B\left(1-\frac{R}{M}\right) - d_{SB}(R) + 1\right]\right\}$$

for $\frac{1}{M\log(2)} \le \beta < \infty$.

Furthermore, the SNR random coding exponent of the associated BICM channel satisfies the same lower bounds.

Corollary 1 *The SNR reliability function of the block-fading channel with input $\mathcal{X}$ and of the associated BICM channel is given by $d_B^\star(R) = d_{SB}(R)$ for all $R \in (0, M]$, except for the N_B discontinuity points of $d_{SB}(R)$, i.e., for the values of R for which $N_B(1 - R/M)$ is an integer.*

These results state that for large block length, the optimal diversity order is given by the Singleton bound on the block diversity, and that it is achievable by random codes, for a block length that grows like $L = \beta\log\rho$ for large enough β.

3. BLOCKWISE CONCATENATED CODES

Figure 1 shows the proposed encoder structure for $\mathcal{M}(\mathcal{C},\mu,\mathcal{X})$ that we refer to as *Blockwise Concatenated Coding* (BCC). The binary linear code $\mathcal{C}$ is formed by the concatenation of a binary linear outer code $\mathcal{C}^O$ of rate r_O and block length $N_B L_\pi$, partitioned into N_B blocks of length L_π. The blocks are separately interleaved by the permutations $(\pi_1, \ldots, \pi_{N_B})$ and the result is fed into the N_B encoders for the inner code $\mathcal{C}^I$ of rate r_I and length $L_B = LM$. Thus, the total length of $\mathcal{C}$ is $N_B L_B$ (binary symbols). Finally, the output of each component inner code is mapped onto a sequence of signals in $\mathcal{X}$ by the one-to-one symbol mapper μ. We denote by K the number of information bits per codeword. In particular, the codes considered make use of bit-interleaving between the inner encoder and the mapper [5], denoted in Figure 1 by the permutations $(\pi_1^\mu, \ldots, \pi_{N_B}^\mu)$. However, we hasten to say that mapping through interleavers is not necessary for the construction and more general mappings could be envisaged. The rate of the resulting blockwise concatenated code is $R = r_O r_I M$.

When the outer code is a simple repetition code of rate $r_O = 1/N_B$ and the inner codes are rate-one accumulators [6], the resulting BCC is referred to as Repeat and *Blockwise* Accumulate (RBA) code. Since interleavers and inner encoding are performed on a blockwise basis, the block diversity of the concatenated code coincides with the block diversity of the outer code. For example, a RBA code has always full diversity $d_B = N_B$. When both outer and inner codes are convolutional codes, we will refer to the resulting structure as *blockwise* concatenated convolutional codes (BCCC).

Practical decoding of BCC resorts to the well-known BP iterative decoding algorithm over the code graph [7]. In particular, when either $\mathcal{C}^O$ or $\mathcal{C}^I$ are convolutional codes, the well-known forward-backward decoding algorithm is used over the subgraph representing the corresponding trellis [8]. In the case of non-binary modulations, we consider a suboptimal decoder that consists of producing, for each received symbol, the posterior probabilities of the binary coded symbols in its label (defined by the symbol mapper μ), and then feeding these probabilities to the decoder for the binary code $\mathcal{C}$ over the resulting binary-input continuous-output channel. We nickname such decoder as BICM-ML decoder. BICM-ML decoding is known to yield near-optimal performance when coupled with Gray mapping.

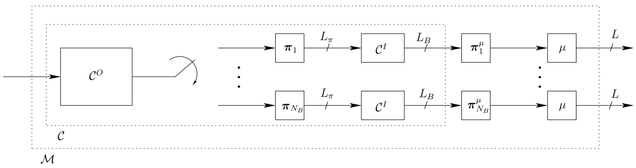

Figure 1: The general encoder for Blockwise Concatenated Coding.

4. GOODNESS OF BCCs

We say that a code ensemble over $\mathcal{X}$ is good if, for block length $L \to \infty$, its WER shows a fixed SNR gap to outage probability, asymptotically independent of L. In this section we give an explicit sufficient condition for code goodness in terms of the asymptotic exponential growth rate function [9] of the multivariate weight enumerator of explicit code ensembles.

Characterizing the goodness of a given code ensemble is non-trivial, as illustrated by the following argument. A code ensemble $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ such that, for all sufficiently large L, a randomly generated member in the ensemble attains the Singleton bound with probability 1 is a good candidate for code goodness. However, this condition is neither necessary nor sufficient. For example, the ensemble $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ considered in Theorem 1 has a small but non-zero probability that a randomly selected member is not blockwise MDS, nevertheless it attains the optimal SNR exponent provided that L grows faster than $\log \rho$, and hence it is good. On the contrary, the ensemble of random BCCs with given outer and non-trivial inner encoders and the ensemble of blockwise partitioned CCs (i.e., BCCs with convolutional outer encoder and rate-1 identity encoder considered in [10, 11]) that can be seen as BCCs with convolutional outer encoder and trivial (identity) inner encoder, attain the Singleton bound with probability 1 provided that the outer code is blockwise MDS. Nevertheless, simulations show that while the WER of general BCCs with recursive inner encoder is almost independent of the block length, the WER of CCs grows roughly linearly with the block length. For example, Fig.2 shows the WER for fixed SNR versus the information block length K, for the ensemble of $R = 1/4$ RBA codes and the standard 64-states CCs with generators $(135, 135, 147, 163)_8$ mapped over $N_B = 4$ blocks, and of $r = 1/2$ BCCs (with outer convolutional encoder $(5, 7)_8$ and inner accumulators) and the 64-states CCs mapped over $N_B = 8$ blocks optimized in [11] with generators $(103, 147)_8$ for the block-fading channel. The different behavior of the WER as a function of the block length for the two ensembles is evident.

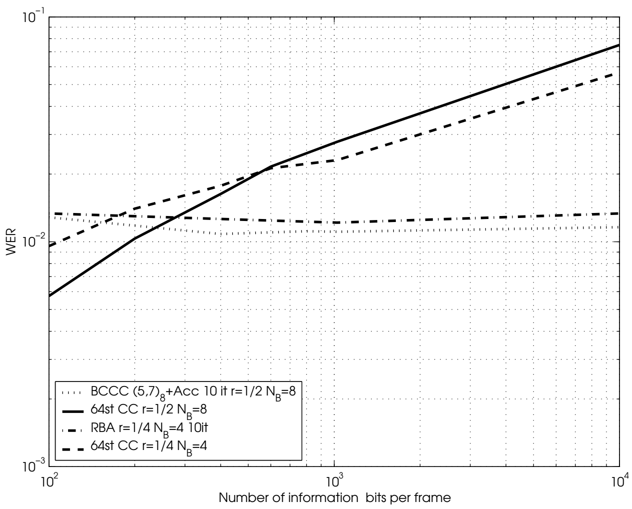

Figure 2: WER vs. information block length at $E_b/N_0 = 8$dB for binary BCC, RBA and trellis terminated CCs obtained by simulation (10 BP decoding iterations for the BCCs and ML Viterbi decoding for the CCs).

We focus first on codes over the BPSK modulation. Therefore, in this case $L = L_B$. Let $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_{N_B}) \in [0, 1]^{N_B}$ be the vector of normalized Hamming weights per block. The asymptotic exponential growth rate function [9] of the multivariate weight enumerator is defined by

$$a(\boldsymbol{\omega}) \triangleq \lim_{\epsilon \to 0} \lim_{L_B \to \infty} \frac{1}{L_B} \log \left| \mathcal{S}_{\epsilon}^{L_B}(\boldsymbol{\omega}) \right| \tag{3}$$

where $\mathcal{S}_{\epsilon}^{L_B}(\boldsymbol{\omega})$ is the set of codewords in the length-L_B ensemble with Hamming weights per block satisfying

$$|w_b / L_B - \omega_b| \leq \epsilon, \quad b = 1, \ldots, N_B \tag{4}$$

We have the following results:

Theorem 2 *Consider an ensemble of codes $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ of rate R, where $\mathcal{X}$ is BPSK, transmitted over a block-fading channel with N_B blocks. Let $a(\boldsymbol{\omega})$ be the asymptotic exponential growth rate function of the ensemble* *multivariate weight enumerator. For $1 \le k \le N_B$, let $\mathcal{W}(N_B, k) \in \mathbb{F}_2^{N_B}$ denote the set of binary fading vectors with Hamming weight not smaller than $N_B - k + 1$, and define $\widehat{s}$ to be the infimum of all $s \ge 0$ such that*

$$\inf_{\mathbf{x} \in \mathcal{W}(N_B, d_{SB}(R))} \inf_{\boldsymbol{\omega} \in [0,1]^{N_B}} \left\{ s \sum_{b=1}^{N_B} x_b \, \omega_b - a(\boldsymbol{\omega}) \right\} > 0 \tag{5}$$

If $\widehat{s} < \infty$, then the code ensemble is good.

As far as higher order coded modulations are concerned, we have the following

Corollary 2 *Consider an ensemble of codes $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ of rate R, where $\mathcal{X}$ is a complex signal set of size 2^M, transmitted over a block-fading channel with N_B blocks, where modulation is obtained by (random) bit-interleaving and decoding by the BICM-ML decoder. If the underlying ensemble of binary codes (i.e., mapping the binary symbols directly onto BPSK) is good, then the ensemble $\mathcal{M}(\mathcal{C}, \mu, \mathcal{X})$ is good.*

The above results (and the proofs in [4]) reveal that the error probability of good codes in the regime where both the block length and the SNR are large is dominated by the event that more than $d_{SB}(R)$ fading components are *small* (in the sense of the proof of Theorem 2). This is precisely the same behavior of the information outage probability for the rate R and discrete signal set $\mathcal{X}$. On the contrary, when less than $d_{SB}(R)$ fading components are small, the code projected over the significant fading components has a finite ML decoding threshold (with probability 1). Therefore, apart from some SNR gap, its error probability vanishes for all such fading realizations. It is also intuitively clear that, due to this sharp threshold behavior, hitting the SNR transition region (known as "waterfall") for which the error probability is non-vanishing even if the fading has less than $d_{SB}(R)$ small components is an event of small probability. This partially explains why BP iterative decoding performs very close to ML in block-fading channels and why more refined bounding techniques such as the tangential-sphere bound do not provide almost any improvement [12, 4]. In fact, it is well-known that BP and ML perform similarly on both the high-error probability region (below the ML decoding threshold) and in the low-error probability region (above the iterative decoding threshold). The gap between the ML and the iterative decoding thresholds seems to play a negligible role in the block-fading channel, for ensembles of good codes. The sharper and sharper transition between the below-threshold and above-threshold regimes of random-like concatenated codes is referred to as *interleaving gain* in [13, 14]. We argue that code goodness of BCCs in block-fading channels is another manifestation of interleaving gain, even if on such channel no waterfall behavior is observed.

It can also be shown that the ensemble of trellis terminated CCs of increasing block length considered in [10, 11] does not satisfy the condition of Theorem 2.

Numerical verification of Theorem 2 is needed for a specific code ensemble. In particular, one has to show that

$$\sup_{\mathbf{x} \in \mathcal{W}(N_B, d_{SB}(R))} \sup_{\boldsymbol{\omega} \in [0,1]^{N_B}} \frac{a(\boldsymbol{\omega})}{\sum_{b=1}^{N_B} x_b \omega_b} < \infty \tag{6}$$

Supported by simulations and by explicit calculation of the multivariate weight enumerator for RBAs (see [6] and [12, 4] for details), we conjecture this is true for the family of random BCCs with MDS outer code and inner recursive encoders.

Combining the limiting before average technique introduded by Malkamaki and Leib (M&L) in [15] with the Bhattacharyya union bound for a code over the BPSK signal set we can write

$$P_e(\rho) \le \mathbb{E}\left[\min\left\{1, \sum_{\boldsymbol{\omega} \in [0,1]^{N_B}} e^{-L_B\left(\rho \sum_{b=1}^{N_B} \gamma_b \omega_b \, - \, F(\boldsymbol{\omega})\right)}\right\}\right]$$

where $F(\boldsymbol{\omega}) \triangleq \frac{1}{L_B} \log A_{\mathbf{w}}$ is the fixed length growth rate of the multivariate weight enumerator $A_{\mathbf{w}}$ and $\mathbf{w} = (\omega_1 L_B, \ldots, \omega_{N_B} L_B) \in \mathbb{N}^{N_B}$ is the weight vector.

Since $\min(1, f(x))$ is continuous in x for continuous f and $\min(1, f(x)) \le 1$, we can apply the dominated convergence theorem [16] and write,

$$\begin{aligned}
&\lim_{L_B \to \infty} \mathbb{E}\left[\min\left\{1, \sum_{\boldsymbol{\omega} \in [0,1]^{N_B}} e^{-L_B\left(\rho \sum_{b=1}^{N_B} \gamma_b \omega_b \, - \, F(\boldsymbol{\omega})\right)}\right\}\right] \\
&= \mathbb{E}\left[\min\left\{1, \lim_{L_B \to \infty} \sum_{\boldsymbol{\omega} \in [0,1]^{N_B}} e^{-L_B\left(\rho \sum_{b=1}^{N_B} \gamma_b \omega_b \, - \, F(\boldsymbol{\omega})\right)}\right\}\right] \\
&= \mathbb{E}\left[\min\left\{1, \lim_{L_B \to \infty} \sum_{\boldsymbol{\omega} \in [0,1]^{N_B}} e^{-L_B\left(\rho \sum_{b=1}^{N_B} \gamma_b \omega_b \, - \, a(\boldsymbol{\omega})\right)}\right\}\right]
\end{aligned} \tag{7}$$

The factor multiplying L_B in the exponent of the RHS of (7) is positive for a given channel realization $\boldsymbol{\gamma}$ if

$$\widehat{\rho} \triangleq \max_{\boldsymbol{\omega} \in [0,1]^{N_B}} \frac{a(\boldsymbol{\omega})}{\sum_{b=1}^{N_B} \gamma_b \omega_b} < \rho \tag{8}$$

Conditioning with respect to $\boldsymbol{\gamma}$ we have that, in the limit of large L_B, $P_e(\rho|\boldsymbol{\gamma}) \to 0$ if $\rho > \widehat{\rho}$ while $P_e(\rho|\boldsymbol{\gamma}) \leq 1$ otherwise. It follows that in the limit for $L_B \to \infty$ the M&L Bhattacharyya bound takes on the form

$$P_e(\rho) \leq \Pr\left(\max_{\boldsymbol{\omega}\in[0,1]^{N_B}} \frac{a(\boldsymbol{\omega})}{\sum_{b=1}^{N_B} \omega_b \gamma_b} \geq \rho \right). \qquad (9)$$

Equation (9) illustrates that the asymptotic performance is given by the distribution of the decoding threshold (in this case the ML union bound threshold). In order to obtain the BP decoding asymptotic performance, we should characterize the distribution of the BP decoding threshold. Unfortunately this can be computationally very expensive, since we need Montecarlo averaging, and for every channel realization we must compute one density evolution. Therefore, in this case, the need for extremely computationally efficient density evolution is evident.

As an example, in Fig. 3 we show the asymptotic WER for the RBA ensemble of rate 1/2 with BPSK modulation, over a channel with $N_B = 2$ fading blocks. The asymptotic WER is computed via the asymptotic Bhattacharyya M&L bound given by (9). Simulations (BP iterative decoder) for information block lengths $K = 100, 1000$ and 10000 are shown for comparison. This figure clearly shows that the WER of these codes becomes quickly independent of the block length and shows fixed gap from the outage probability.

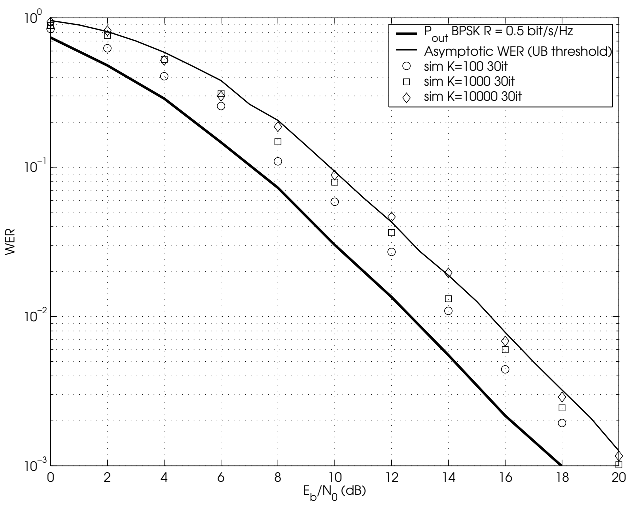

Figure 3: Asymptotic error probability (9) for a binary rate $r = 1/2$ RBA code mapped over $N_B = 2$ fading blocks and corresponding BP decoding simulation with 30 iterations and $K = 100, 1000$ and 10000.

In order to illustrate the goodness of BCCs with BICM and high-order modulations, Fig. 4 shows the asymptotic WER of an RBA code of rate $R = 2$ bit/complex dimension with 16-QAM modulation over $N_B = 2$ fading blocks. The asymptotic WER can be derived in a similar way by using the asymptotic BICM Bhattacharyya M&L bound given by

$$P_e(\rho) \leq \Pr\left(\max_{\boldsymbol{\omega}\in[0,1]^{N_B}} \frac{a(\boldsymbol{\omega})}{\sum_{b=1}^{N_B} \omega_b \zeta_b} \geq \rho \right), \qquad (10)$$

where $\zeta_b = -\frac{1}{\rho} \log B_b(\rho, \mu, \mathcal{X})$,

$$B_b(\rho,\mu,\mathcal{X}) = \mathbb{E}_{m,a,x,Z}\left[\sqrt{ \frac{\displaystyle\sum_{x'\in\mathcal{X}_{\bar{a}}^m} e^{-|\sqrt{\rho\gamma_n}(x-x')+Z|^2}}{\displaystyle\sum_{x'\in\mathcal{X}_a^m} e^{-|\sqrt{\rho\gamma_n}(x-x')+Z|^2}} } \right]$$

is the Bhattacharyya factor of the BICM channel associated to the b-th fading block, with SNR $\gamma_b\rho$, and $\mathcal{X}_a^m$ is the set of constellation points for which the m-th label position has content $a \in \{0,1\}$. Simulations (BP iterative decoder) for information block lengths $K = 100, 1000$ and 10000 are shown for comparison, and we can observe the same effect as for the BPSK case.

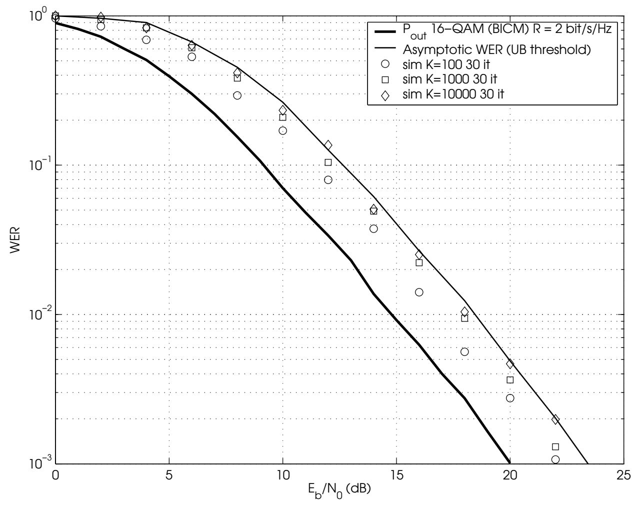

Figure 4: Asymptotic error probability (10) for a rate $R = 2$ RBA code mapped over $N_B = 2$ fading blocks with 16-QAM (BICM) and corresponding BP decoding simulation with 30 iterations for $K = 100, 1000$ and 10000.

5. CONCLUSIONS

In this paper we have studied the asymptotic performance of blockwise concatenated coded modulation in the block-fading channel. By using asymptotic weight enumerator techniques, we have shown that for large block length, the error probability is given by the distribution of the decoding threshold. We have also argued that, as opposed to ergodic channels, in the block-fading channel, belief-propagation and maximum-likelihood decoding perform very similar. This represents a remarkable improvement with respect to standard trellis-based codes with a wide range of applications.

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