# New Space-Time Trellis Codes for Slow Fading Channels

Yi Hong
Institute for Telecommunications Research,
University of South Australia,
Mawson Lakes SA 5095, Australia
Email: Yi.Hong@unisa.edu.au

Albert Guillén i Fàbregas
Institute for Telecommunications Research,
University of South Australia,
Mawson Lakes SA 5095, Australia
Email: albert.guillen@unisa.edu.au

***Abstract*— New space-time trellis codes with 4-PSK, 8-PSK and 16-QAM for two transmit antennas in slow fading channels are proposed in this paper. The codes are designed specifically to minimize the frame error probability. The performance of the proposed codes with various memory orders and receive antennas is evaluated by simulation. It is shown that the proposed codes outperform previously known codes [1].**

## I. Introduction

Space-time trellis coding (STTC) techniques [1] have been proposed to achieve both diversity and coding gains on multi-input multi-output (MIMO) fading channels by combining multiple transmit antennas and coding with higher level modulation schemes. In [1], rank and determinant criteria (RDC) were proposed to maximize both diversity and coding gains of STTCs over slow fading channels. Several efforts have been dedicated to further maximize the coding gain using RDC [2, 3]. In [4], the determinant criterion was strengthened by analysis of the role of the Euclidean distance. In [5], STTCs are designed based on either RDC or EDC depending on the diversity gain of the system. Based on above criteria, some improved 4-PSK and 8-PSK STTCs are proposed through exhaustive computer search [5, 6].

For high order constellations such as 16-QAM, a special rank criterion is proposed in [7], the core of which is a sufficient condition to select full rank codes without exhaustive computer search. This simplifies the code design. However, in [7], the coding gain is not optimized. In [8], better 16-QAM STTCs are found by optimizing the coding gain based on EDC.

A common feature of all above code design criteria is to minimize the worst case pairwise error probability (PWEP). To further improve code performance, the design criteria proposed in [9] attempt to minimize the worst case frame error probability, which is a function of the distance spectrum of the code. The distance spectrum is an enumeration of all the possible product measures (non-zero determinants) with their relative weights [10]. In [9], only a 4-state 4-PSK STTC was designed.

In this paper, we consider the design guidelines, which aim to minimize the truncated union bound on frame error rate (FER) by taking into account the first three terms. The PWEP terms depend on determinants and the associate weights for the systems with low and moderate diversity gains. This is similar to the approach taken in [11], where the PWEP terms depend on the Euclidean distances instead for moderate diversity gains. In our design, we construct three complete sets of 4-PSK, 8-PSK and 16-QAM STTCs for two transmit antennas over slow fading channels. Through simulations, it is shown that the new codes outperform previously known codes.

The rest of the paper is organized as follows. Section II introduces the system model and code design criteria. Section III introduces STTC encoder structures for PSK and 16-QAM, respectively. In Section IV, new 4-PSK, 8-PSK and 16-QAM STTCs are presented together with simulation results. Finally, conclusions are drawn in Section V.

## II. System Model and Code Design Criteria

The following notations are used in the paper: $T$ denotes transpose and † denotes transpose conjugate. The symbol $\oplus_M$ and and $\ominus_M$ denotes modulo $M$ addition and substraction. Superscripts $I$ and $Q$ denote the real and imaginary parts of a complex number. Let $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ denote the ring of the integers $\oplus_4$ and let $\mathbb{Z}_4[j]$ be the ring of Gaussian integers modulo 4, where each element $z \in \mathbb{Z}_4[j]$ has $\left\{z = z^I + jz^Q : z^I, z^Q \in \mathbb{Z}_4\right\}$ and $j^2 = -1$.

We consider a space-time coding system with $n_T$ transmit and $n_R$ receive antennas over slow fading channels. The received signal matrix $\mathbf{Y} \in \mathbb{C}^{n_R \times L}$ is given by

$$\mathbf{Y} = \sqrt{E_s}\mathbf{H}\mathbf{X} + \mathbf{N}, \tag{1}$$

where $L$ is the frame length, $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_t, ..., \mathbf{x}_L] \in \mathbb{C}^{n_T \times L}$ is the transmitted signal matrix, where $\mathbf{x}_t$=$[x_t^1, ..., x_t^{n_T}]^T$, $\mathbf{H} = [\mathbf{h}_1, ..., \mathbf{h}_{n_T}] \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, which is constant during a frame and varies from one frame to another independently. In (1), $\mathbf{N} \in \mathbb{C}^{n_T \times L}$ is a matrix of the complex white Gaussian noise samples i.i.d $\sim \mathcal{N}_{\mathbb{C}}(0, N_0)$, and $\frac{E_s}{N_0}$ is the average signal to noise ratio (SNR) per transmit antenna. The elements of $\mathbf{H}$ are assumed to be i.i.d circularly symmetric Gaussian random variables $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. The channel is assumed to be known at the receiver.

Assume that a codeword $\mathbf{X}$ is transmitted, the maximum-likelihood receiver might decide erroneously in favor of

[1]This work has been supported by the Australian Research Council (ARC) Grants DP0344856 and DP0558861.

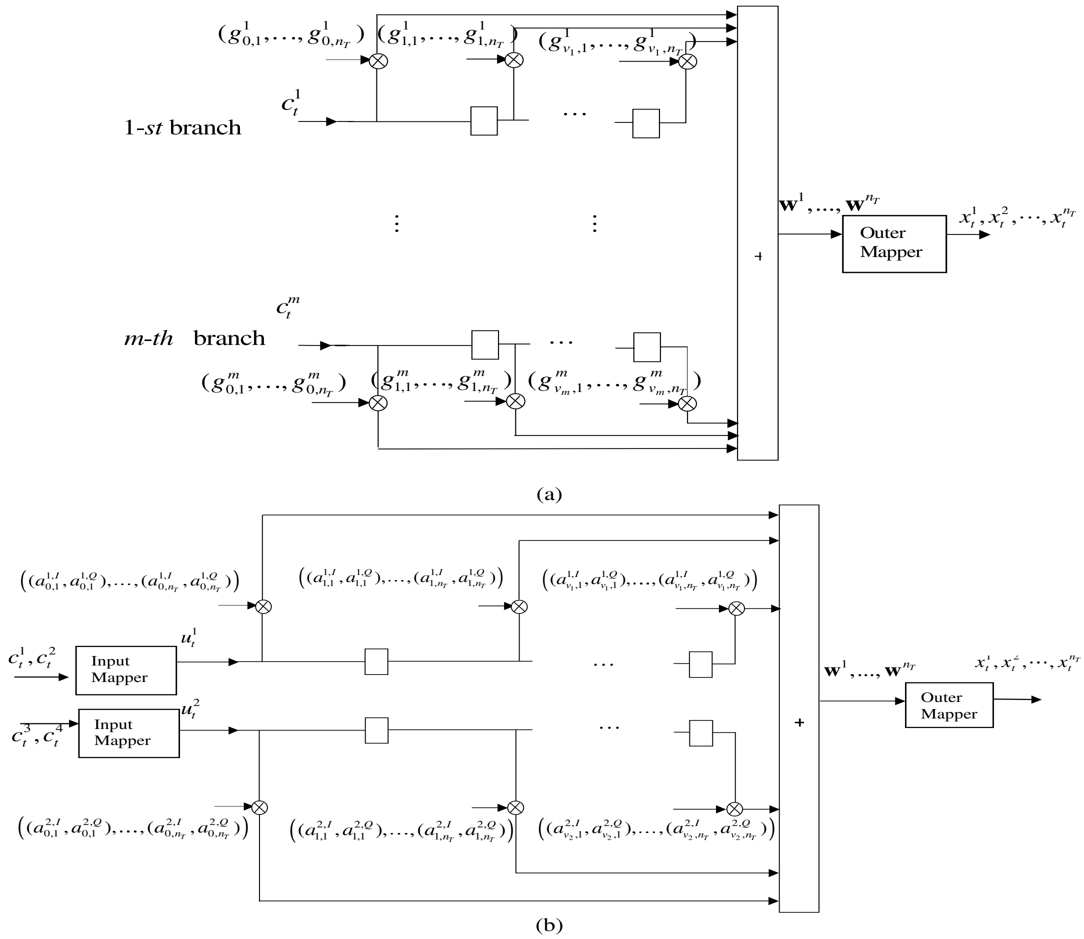


Fig. 1. *M*-PSK (a) and 16-QAM (b) STTC encoder with $n_T$ transmit antennas.

another codeword $\hat{\mathbf{X}} = [\hat{\mathbf{x}}_1, ..., \hat{\mathbf{x}}_t, ..., \hat{\mathbf{x}}_L]$, where $\hat{\mathbf{x}}_t = [\hat{x}_t^1, ..., \hat{x}_t^{n_T}]^T$. Let $r$ denote the rank of the codeword difference matrix $\mathbf{B}(\mathbf{X}, \hat{\mathbf{X}}) = \mathbf{X} - \hat{\mathbf{X}}$, and $\lambda_k$ be the eigenvalues of the codeword distance matrix $\mathbf{A} = \mathbf{B}\mathbf{B}^\dagger$. Here we only consider the STTCs with full rank $r = n_T$. Defining $d$ as the determinant of the codeword distance matrix $\mathbf{A}$, we have $d = \left(\prod_{k=1}^{r} \lambda_k\right)^{-n_R}$. Let $\mathcal{D}$ be a set of all possible determinants of the codeword distance matrix $\mathbf{A}$, namely product measures. Then the union bound on the FER yields [9]

$$P_{FER} \leq \left(\sum_{d\in\mathcal{D}} \bar{N}(d) d^{-n_R}\right) \left(\frac{E_s}{N_0}\right)^{-rn_R} \tag{2}$$

with

$$\bar{N}(d) = \frac{1}{m^v} \sum_{p=p_{min}}^{p_{max}} \frac{N(d,p)}{(2)^{mp}}, \tag{3}$$

where $m$ is the number of bits per symbol for PSK and QAM, $p_{min}$=$\lfloor v/b + 1 \rfloor$ is the minimum length of trellis paths for a simple error event [10], $v$ is the memory order of the encoder, and $b = \log_2 M$ and $\log_4 M$ for $M$-PSK and $M$-QAM, respectively.

In (3), $p_{max}$ is the maximum length of the trellis path that we take into account [5], $N(d,p)$ is the number of error events which have the determinant $d$ over trellis paths with the length $p$, and $\bar{N}(d)$ is the associated weight [10].

Defining

$$\eta(p_{max}) = \frac{1}{m^v} \sum_{d\in\mathcal{D}} \left\{ \sum_{p=p_{min}}^{p_{max}} \frac{N(d,p)}{(2)^{mp}} \right\} d^{-n_R}, \tag{4}$$

the code design criteria for STTCs are formulated as follows:

1) Diversity gain: The codeword distance matrix $\mathbf{A}$ has to be full rank for maximizing the diversity gain;
2) Coding gain: To optimize the coding gain, the term $\eta(p_{max})$ has to be minimized over all the possible error events in the trellis diagram.

## III. STTC Encoders

In this Section, we introduce two different STTC encoder structures for PSK and QAM, respectively. Both of the trellis

TABLE I
4-PSK STTCs

| $v$ | code | Generator Coefficients $\mathbf{g}^1,\mathbf{g}^2$ | Determinants $d_1, d_2, d_3$ | Weight $\bar{N}(d_i)$, $i=1,2,3$ | $\eta(p_{max})$ $n_R=2$ | SNR (dB) FER=$10^{-4}$ $n_R=1$ | SNR (dB) FER=$10^{-4}$ $n_R=2$ |
|---|---|---|---|---|---|---|---|
| 2 | [TSC] | [(0,2),(2,0)], [(0,1),(1,0)] | (4, 12, 16) | (2, 4, 1) | 1.6e-1 | 31 | 18.88 |
| | [YCVF] | [(0,2),(1,0)], [(2,2),(0,1)] | (8, 12, 20) | (3,2.12,3) | -- | 31 | -- |
| | | [(0,2),(1,2)], [(2,3),(2,0)] | (4, 8, 12) | (0.25, 2, 0.5) | 5.0e-2 | -- | 18.3 |
| | [JL] | [(1,2),(2,2)], [(2,1),(0,2)] | (8, 12, 16) | (1.5, 2.12, 2) | 4.2e-2 | 30.96 | 18.15 |
| | New | [(0,2),(2,1)], [(2,2),(3,2)] | (8, 12, 16) | (1.5, 2.12, 1) | 4.2e-2 | 30.95 | 17.96 |
| 3 | [TSC] | [(0,2),(2,0)], [(0,1),(1,0),(2,2)] | (12, 16, 20) | (2, 1, 1) | 2.0e-2 | 30.2 | 17.5 |
| | [YCVF] | [(0,2),(2,0)], [(2,1),(1,2),(0,2)] | (16, 20, 28) | (1,5,1) | -- | 29.91 | -- |
| | | [(2,2),(2,1)], [(2,0),(1,2),(0,2)] | (8, 12, 16) | (0.5, 1, 1) | 1.9e-2 | -- | 17.35 |
| | [LP] | [(0,2),(2,1)], [(2,0),(1,3),(0,2)] | (16, 20, 28) | (1, 3.5, 0.7) | -- | 29.8 | -- |
| | | [(0,2),(2,2)], [(2,1),(1,1),(0,2)] | (12, 16, 20) | (0.25, 1, 1) | 8e-3 | -- | 16.84 |
| | New | [(1,2),(2,0)], [(2,0),(3,1),(0,2)] | (16, 20, 28) | (1, 0.35, 1) | 6e-3 | 29.69 | 16.6 |
| 4 | [TSC] | [(0,2),(2,0),(0,2)], [(0,1),(1,2),(2,0)] | (12, 20, 28) | (1, 1.5, 3.88) | 1.6e-2 | 29 | 16.75 |
| | [YCVF] | [(0,2),(1,2),(2,2)], [(2,0),(1,1),(0,2)] | (32, 36, 44) | (2.25, 2.35, 3) | -- | 28.5 | -- |
| | | [(1,2),(1,3),(3,2)], [(2,0),(2,2),(2,0)] | (8, 16, 24) | (0.25, 0.063, 2.56) | 1.3e-2 | -- | 16.6 |
| | New | [(2,2),(2,1),(2,0)], [(0,2),(3,2),(2,2)] | (32, 36, 44) | (2, 2.53, 3.12) | 5.5e-3 | 28.4 | 15.75 |
| 5 | [YCVF] | [(2,0),(2,3),(0,2)],[(2,2),(1,0),(1,2),(2,2)] | (36, 40, 44) | (0.25, 1.53, 1.88) | -- | 28.2 | -- |
| | | [(0,2),(2,3),(1,2)],[(2,2),(1,2),(2,3),(2,0)] | (20, 24, 28) | (0.25, 0.25, 0.5) | 1.7e-3 | -- | 15.85 |
| | New | [(0,2),(2,1),(2,0)], [(2,2),(1,0),(1,2),(2,2)] | (36, 40, 44) | (0.006, 1.25, 1.38) | 1.5e-3 | 28 | 15.5 |
| 6 | [YCVF] | [(1,2),(2,2),(0,3)(2,0)],[(2,0),(2,0),(1,3),(0,2)] | (40, 48, 52) | (0.01, 0.3, 0.4) | -- | 26.8 | -- |
| | | [(0,2),(3,1),(3,3)(3,2)],[(2,2),(2,2),(0,0),(2,0)] | (32, 40, 44) | (0.4, 0.5, 0.1) | 7.5e-4 | -- | 15.5 |
| | New | [(2,2),(0,1),(2,0)(0,2)],[(0,2),(1,0),(1,2),(2,2)] | (48, 52, 56) | (0.3, 0.125, 1.5) | 6.5e-4 | 26.6 | 15.25 |

encoders have the rate of 1/2.

### A. $M$-PSK STTC Encoder

An $M$-PSK STTC encoder with memory order $v$ and $n_T$ transmit antennas is shown in Fig. 1 (a). The $M$-PSK STTC encoder consists of an $m$-branch shift register with total memory order $v$. At time $t$, $m$ binary inputs $c_t^i$, $i = 1, 2, \ldots, m$, are fed into the $m$ branches. The memory order of the $i$-th branch, $v_i$, is given by

$$v_i = \left\lfloor \frac{v+i-1}{b} \right\rfloor, \tag{5}$$

where $\lfloor x \rfloor$ denotes the maximum integer not larger than $x$.

The $m$ streams of input bits are simultaneously passed through their respective shift register branches and multiplied by the coefficient vectors,

$$\begin{aligned}
\mathbf{g}^1 &= [(g_{0,1}^1, g_{0,2}^1, \ldots, g_{0,n_T}^1), \ldots, (g_{v_1,1}^1, g_{v_1,2}^1, \ldots, g_{v_1,n_T}^1)] \\
&\vdots \\
\mathbf{g}^m &= [(g_{0,1}^m, g_{0,2}^m, \ldots, g_{0,n_T}^m), \ldots, (g_{v_m,1}^m, g_{v_m,2}^m, \ldots, g_{v_m,n_T}^m)],
\end{aligned}$$

where $g_{q_i,k}^i \in \{0, 1, \ldots, M-1\}$, $i = 1, 2, ..., m$, $q_i = 0, 1, 2, ..., v_i$, $k = 1, 2, ..., n_T$. The encoder output $w_t^k$, $t = 1, ..., L$, $k = 1, ..., n_T$, can be computed as

$$w_t^k = \left( \sum_{i=1}^{m} \sum_{q_i=1}^{v_i} \left( g_{q_i,k}^i c_{t-q_i}^k \right) \right) \mod M. \tag{6}$$

These outputs are mapping into $x_t^k$ in an $M$-PSK constellation. The STTC encoder can also be described in generator polynomial format. The binary input stream $\mathbf{c}^i$ can be represented as

$$\mathbf{c}^i(D) = c_0^i + c_1^i D + c_2^i D^2 + \cdots + c_t^i D^t + \cdots, \tag{7}$$

where $D$ represents a unit delay operator. The generator matrix for antenna $k$ can be represented as

$$\mathbf{G}_k(D) = \begin{bmatrix} \mathbf{G}_k^1(D) \\ \mathbf{G}_k^2(D) \\ \vdots \\ \mathbf{G}_k^m(D) \end{bmatrix}, \tag{8}$$

where

$$\mathbf{G}_k^i(D) = g_{0,k}^i + g_{1,k}^i D + \cdots + g_{v_i,k}^i D^{v_i}, \tag{9}$$

is the $i-$th branch generator polynomial for the transmit antenna $k$. The coded symbol sequence transmitted from antenna $k$ is given by

$$\mathbf{w}^k(D) = \left( \sum_{i=1}^{m} \mathbf{c}^i(D) \mathbf{G}_k^i(D) \right) \mod M. \tag{10}$$

### B. 16-QAM STTC Encoder

A 16-QAM STTC encoder with memory order $v$ and $n_T$ transmit antennas is shown in Fig. 1 (b). At time $t$, the input mapper converts 4 input bits $\left(c_t^1, c_t^2, c_t^3, c_t^4\right)$ into 2 components $u_t^1$ and $u_t^2$ through natural mapping, where $u_t^1, u_t^2 \in \mathbb{Z}_4$. The two components go through two-branch shift register with total memory order $v$. The memory order of the $i$-th branch, $v_i, i = 1, 2$, is given in (5). The two streams of the components are multiplied by the coefficient vectors, which are given in Eq. (11), where $a_{q_i,k}^{i,I}, a_{q_i,k}^{i,Q} \in \mathbb{Z}_4$, and $a_{j_k,i}^{k,I} + j a_{j_k,i}^{k,Q} \in \mathbb{Z}_4[j]$, $i = 1, 2$, $q_i = 0, 1, 2, ..., v_i$. The encoder output $w_t^k$, $t = 1, ..., L$, $k = 1, ..., n_T$ can be computed as

$$w_t^k = \sum_{i=1}^{2} \sum_{q_i=1}^{v_i} a_{q_i,k}^{i,I} u_{t-q_i}^k + j \sum_{i=1}^{2} \sum_{q_i=1}^{v_i} a_{q_i,k}^{i,Q} u_{t-q_i}^k \mod 4, \tag{12}$$

$$\begin{aligned} \mathbf{g}^1 &= \left[\left(\left(a_{0,1}^{1,I},a_{0,1}^{1,Q}\right),...,\left(a_{0,n_T}^{1,I},a_{0,n_T}^{1,Q}\right)\right),...,\left(\left(a_{v_1,1}^{1,I},a_{v_1,1}^{1,Q}\right),...,\left(a_{v_1,n_T}^{1,I},a_{v_1,n_T}^{1,Q}\right)\right)\right] \\ \mathbf{g}^2 &= \left[\left(\left(a_{0,1}^{2,I},a_{0,1}^{2,Q}\right),...,\left(a_{0,n_T}^{2,I},a_{0,n_T}^{2,Q}\right)\right),...,\left(\left(a_{v_2,1}^{2,I},a_{v_2,1}^{2,Q}\right),...,\left(a_{v_2,n_T}^{2,I},a_{v_2,n_T}^{2,Q}\right)\right)\right] \end{aligned} \quad (11)$$

where $w_t^k \in \mathbb{Z}_4[j]$. In generator polynomial format, the input components $\mathbf{u}^i, i = 1, 2$, can be represented as

$$\mathbf{u}^i(D) = u_0^i + u_1^i D + u_2^i D^2 + \cdots + u_t^i D^t + \cdots . \quad (13)$$

The generator matrix for antenna $k$ can be represented as

$$\mathbf{G}_k(D) = \begin{bmatrix} \mathbf{G}_k^1(D) \\ \mathbf{G}_k^2(D) \end{bmatrix}, \quad (14)$$

where

$$\mathbf{G}_k^i(D) = \left(a_{0,k}^{i,I} + \cdots + a_{v_i,k}^{i,I} D^{v_i}\right) + j\left(a_{0,k}^{i,Q} + \cdots + a_{v_i,k}^{i,Q} D^{v_i}\right), \quad (15)$$

is the $i-$th branch generator polynomial for the transmit antenna $k$. The coded symbol sequence transmitted from antenna $k$ is given by

$$\mathbf{w}^k(D) = \left(\sum_{i=1}^{2} \mathbf{u}^i(D)\mathbf{G}_k^i(D)\right) \mod 4. \quad (16)$$

The coded symbol sequence is mapped by linear translation mapping, i.e. $x_t^k = w_t^k - (3 + 3j)/2$.

## IV. New STTCs and Simulation Results

In this Section, we present new 4-PSK, 8-PSK and 16-QAM STTCs for two transmit antennas over slow fading channels. The signal-to-noise ratio (SNR) per receive antenna is defined as SNR=$n_T E_s/N_0$. For PSK and 16-QAM signal sets, we assume that each frame consists of 130 symbols and 66 symbols out of each transmit antenna. This corresponds to 260 and 264 information bits. The coding gain of 4-PSK, 8-PSK and 16-QAM STTCs are optimized by taking into account the first three distance spectra in the truncated union bound [10]. The coding gain only provides an estimate of performance due to the fact that the length $p_{max}$ considered in the distance spectrum is significantly less than the frame length. Therefore new 4-PSK, 8-PSK and 16-QAM STTCs are chosen based on both the coding gain and further simulation results.

### A. 4-PSK and 8-PSK STTCs

In the code design for 4-PSK and 8-PSK signal sets, generator coefficients are determined through exhaustive search. Since the encoder structure cannot guarantee geometrical uniformity of the code, the search was based on all possible pairwise error events. In order to reduce the complexity of the code search, we use the determinants of the known codes in [5] as the bench marks. The complexity of the code construction is the same as that for previously known codes [5, 11].

Tables I and II list the new and known 4-PSK and 8-PSK STTCs with bandwidth efficiency 2 bits/sec/Hz and

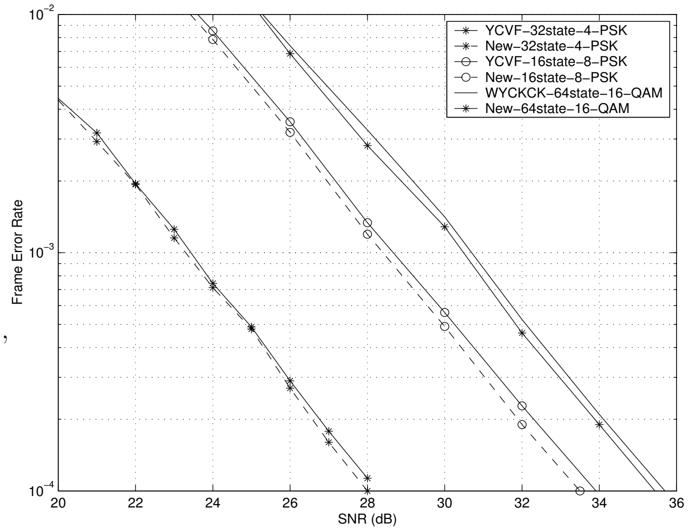


Fig. 2. Performance of 4-PSK, 8-PSK and 16-QAM STTCs, 2Tx 1Rx

3 bits/sec/Hz, respectively. All these codes have full rank of $r = 2$. The codes are described by memory order $(v)$, generator coefficients $(\mathbf{g}^1, \mathbf{g}^2)$, the first three minimum determinants $(d_1, d_2, d_3 \in d)$, the associated weights $(\bar{N}(d_1), \bar{N}(d_2), \bar{N}(d_3))$, the term $(\eta(p_{max}))$ with $p_{max} = 7$. Finally, SNRs at a FER of $10^{-4}$ with $n_R = 1, 2$ are given. Given $n_T = 2$, some known codes in Tables I and II were specifically designed for very low diversity gain, say $n_T n_R < 4$. We only report the corresponding SNRs. We can see that the proposed codes have the lowest SNRs required to achieve the FER of $10^{-4}$, compared to previously known codes.

### B. 16-QAM STTCs

By use of linear translation mapping, a rank criterion is proposed in [7], which is called $\sum_o$ rank criterion. This criterion is to determine the full diversity 16-QAM STTCs in the $\mathbb{Z}_4[j]$ domain rather than in the 16-QAM signal set. It simplifies the code search. A special case of the $\sum_o$ rank criterion is described in [7, Proposition 10]. In our design, both generator matrices $\mathbf{G}_1(D)$ and $\mathbf{G}_2(D)$ in (15) have the special structure described in Proposition 2 [7]. Hence the problem of determining full transmit diversity can be simplified as follows.

Step 1: Based on [7, Proposition 10], we first check the non-singularity of the generator matrix $\mathbf{G}_1(D)$ by determining whether the polynomial $\left(\mathbf{a}_1^{1,I}(D)\,\mathbf{a}_1^{2,Q}(D)\right) \ominus_4 \left(\mathbf{a}_1^{2,I}(D)\,\mathbf{a}_1^{1,Q}(D)\right)$ has at least one odd coefficient. All the possible non-singular generator matrices for $\mathbf{G}_1(D)$ can be also used for $\mathbf{G}_2(D)$. Nevertheless it is redundant to consider

TABLE II
8-PSK STTCs

| $v$ | code | Generator Coefficients $\mathbf{g}^1, \mathbf{g}^2$ | Determinants $d_1, d_2, d_3$ | Weight $\bar{N}(d_i)$, $i=1,2,3$ | $\eta(p_{max})$ $n_R$=2 | SNR (dB) FER=$10^{-4}$ $n_R$=1 | SNR (dB) FER=$10^{-4}$ $n_R$=2 |
|---|---|---|---|---|---|---|---|
| 3 | [TSC] | [(0,4),(4,0)], [(0,2),(2,0)], [(0,1),(5,0)] | (2, 3.37, 4) | (0.5, 0.02, 1) | 0.1893 | 34.45 | 24.8 |
| | [YCVF] | [(0,2),(2,0)], [(0,4),(4,0)], [(0,5),(1,4)] | (4, 4.34, 6.4) | (1,0.03,0.25) | -- | 34.1 | -- |
| | | [(2,1),(3,4)], [(4,6),(2,0)], [(0,4),(4,0)] | (2,3.37,4) | (0.5,0.00165,1) | 0.1876 | -- | 20.8 |
| | New | [(3,4),(4,1)], [(4,0),(0,4)], [(2,0),(0,6)] | (4, 4.34, 6.7) | (1,0.03,0.5) | 0.07 | 34.0 | 20.45 |
| 4 | [TSC] | [(0,4),(4,4)], [(0,2),(2,2)], [(0,1),(5,1),(1,5)] | (3.5,4,4.34) | (0.0156, 1, 0.0625) | 0.8171 | 33.95 | 20.4 |
| | [YCVF] | [(0,4),(4,0)], [(0,2),(2,0)], [(2,0),(6,5),(1,4)] | (4, 4.34, 5.4) | (1,0.006,0.004) | -- | 33.9 | -- |
| | | [(2,4),(3,7)], [(4,0),(6,6)], [(7,2),(0,7),(4,4)] | (0.86, 0.9, 1.1) | (0.002, 0.008, 0.0007) | 0.0132 | -- | 20.6 |
| | New | [(0,6),(6,0)], [(7,4),(4,0)], [(4,5),(2,1),(2,4)] | (4, 4.34, 5.86) | (0.5, 0.0009, 0.001) | 0.0032 | 33.5 | 20.0 |
| 5 | [TSC] | [(0,4),(4,4)],[(0,2),(2,2),(2,2)], [(0,1),(5,1),(3,7)] | (3.51,4,4.34) | (0.008, 0.25, 0.03) | 0.0179 | 33.7 | 19.75 |
| | [YCVF] | [(0,4),(4,4)],[(0,2),(2,2),(0,0)], [(3,5),(2,0),(4,0)] | (7.029,7.5,8.2) | (0.05, 0.06255, 0.094) | -- | 33.6 | -- |
| | | [(0,4),(4,4)],[(0,2),(2,3),(2,2)], [(3,0),(2,2),(3,7)] | (2.69,3.86,4.34) | (0.25,0.0005,0.002) | 0.0347 | -- | 19.5 |
| | New | [(4,0),(2,6)],[(7,5),(6,6),(2,2)], [(7,3),(4,6),(4,4)] | (7.5,8.0,10.8) | (0.0625, 0.0156,0.094) | 0.0022 | 33.5 | 19.4 |

TABLE III
16-QAM STTCs

| $v$ | code | Generator Coefficients $(\mathbf{g}^1)^T, (\mathbf{g}^2)^T$ | Determinants $d_1, d_2, d_3$ | Weight $\bar{N}(d_i)$, $i=1,2,3$ | $\eta(p_{max})$ $n_R$=2 | SNR (dB) FER=$10^{-4}$ $n_R$=1 | SNR (dB) FER=$10^{-4}$ $n_R$=2 |
|---|---|---|---|---|---|---|---|
| 2 | [TSC] | $\begin{pmatrix}(0,0),(1,0)\\(3,2),(0,0)\end{pmatrix},\begin{pmatrix}(0,0),(0,1)\\(2,3),(0,0)\end{pmatrix}$ | (4, 5, 13) | (1.6,4,2) | 0.2718 | 36.4 | 23.6 |
| | [LFT] | $\begin{pmatrix}(3,3),(2,2)\\(2,0),(3,3)\end{pmatrix},\begin{pmatrix}(1,0),(0,2)\\(0,2),(0,1)\end{pmatrix}$ | (1,3,4) | (0.156,0.648,0.312) | 0.2475 | 36.75 | 23.5 |
| | [WYCKCK] | $\begin{pmatrix}(3,0),(0,2)\\(2,1),(1,1)\end{pmatrix},\begin{pmatrix}(1,1),(2,1)\\(0,0),(2,1)\end{pmatrix}$ | (2,6,7) | (0.337,0.0642,0.312) | 0.1395 | 36.9 | 22.6 |
| | New | $\begin{pmatrix}(0,0),(1,2)\\(3,1),(0,2)\end{pmatrix},\begin{pmatrix}(2,1),(0,2)\\(2,2),(2,1)\end{pmatrix}$ | (4, 5, 7) | (0.0158,0.4,0.9) | 0.04 | 36.4 | 22.5 |
| 3 | [WYCKCK] | $\begin{pmatrix}(2,1),(3,3)\\(0,2),(0,1)\end{pmatrix},\begin{pmatrix}(0,2),(0,1)\\(0,3),(2,3)\\(3,0),(2,0)\end{pmatrix}$ | (4, 5, 7) | (0.0132,0.234,0.0148) | 0.0105 | 35.8 | 21.75 |
| | New | $\begin{pmatrix}(1,0),(1,2)\\(1,1),(1,0)\end{pmatrix},\begin{pmatrix}(0,1),(3,1)\\(1,0),(3,1)\\(3,1),(2,3)\end{pmatrix}$ | (5, 6, 7) | (0.234,0.00098, 0.0107) | 0.0096 | 35.4 | 21.6 |

the permutations of $\mathbf{G}_1(D)$ and $\mathbf{G}_2(D)$, since they yield an equivalent code.

Step 2: For all non-equivalent $\sum_o$-coefficient sets $\{\alpha_1, \alpha_2\}$ [7, Proposition 6-8], bsed on [7, Proposition 10], let $\mathbf{G}$ be a linear combination of the generator matrices

$$\begin{aligned} \mathbf{G} &= \alpha_1 \mathbf{G}_1(D) \oplus_4 \alpha_2 \mathbf{G}_2(D) \\ &= \begin{bmatrix} \hat{\mathbf{g}}^{1,I}(D) \\ \hat{\mathbf{g}}^{2,I}(D) \end{bmatrix} + j \begin{bmatrix} \hat{\mathbf{g}}^{1,Q}(D) \\ \hat{\mathbf{g}}^{2,Q}(D) \end{bmatrix} \end{aligned}$$

where $\hat{\mathbf{g}}^{k,I}(D) = \alpha_1 \mathbf{a}_1^{k,I}(D) \oplus_4 \alpha_2 \mathbf{a}_2^{k,I}(D)$, $\hat{\mathbf{g}}^{k,Q}(D) = \alpha_1 \mathbf{a}_1^{k,Q}(D) \oplus_4 \alpha_2 \mathbf{a}_2^{k,Q}(D)$, $k = 1, 2$. All the possible non-singular generator matrices for $[\mathbf{G}_1(D)\mathbf{G}_1(D)]^T$ can be obtained by checking wether the polynomial $\left(\hat{\mathbf{g}}^{1,I}(D)\,\hat{\mathbf{g}}^{2,Q}(D)\right) \ominus_4 \left(\hat{\mathbf{g}}^{1,Q}(D)\,\hat{\mathbf{g}}^{2,I}(D)\right)$ has at least one odd coefficient.

If the generator matrices satisfy both conditions that are described above, the code achieves full transmit diversity [7] and it is further considered in the code selections. Table III lists new and known full diversity 16-QAM STTCs with bandwidth efficiency 4 bits/sec/Hz. The codes are described in the same manner as Tables I and II.

Let $\ell$ be the computation complexity in Step 1. In Step 2, the computation complexity will be $n \times \ell$, where the number of non-equivalent $\sum_o$-coefficient sets is $n = 2^{(m-1)(n_T-1)}2^{n_T} - 1 = 24$ [7]. In our design, code search provides saving costs of $73\%$ and $71\%$ of the full search by simply applying Step 1, for 16-state and 64-state 16-QAM STTCs, respectively. By using Step 2, code search provides another $8.5\%$ and $10.5\%$ saving costs. Hence, the computation complexity for full rank codes in our design will be only $\frac{73\%\times\ell+8.5\%\times n\times\ell}{81.5\%\times n\times\ell} = 14.13\%$ and $\frac{71\%\times\ell+10.5\%\times n\times\ell}{81.5\%\times n\times\ell} = 16.43\%$ of the approach in [8], for 16-state and 64-state 16-QAM STTCs, respectively.

### C. Simulation Results

Fig. 2 compares the performance of the 4-PSK , 8-PSK and 16-QAM STTCs, respectively, for the system with one receive antennas. We can see that the proposed 32-state 4-PSK code

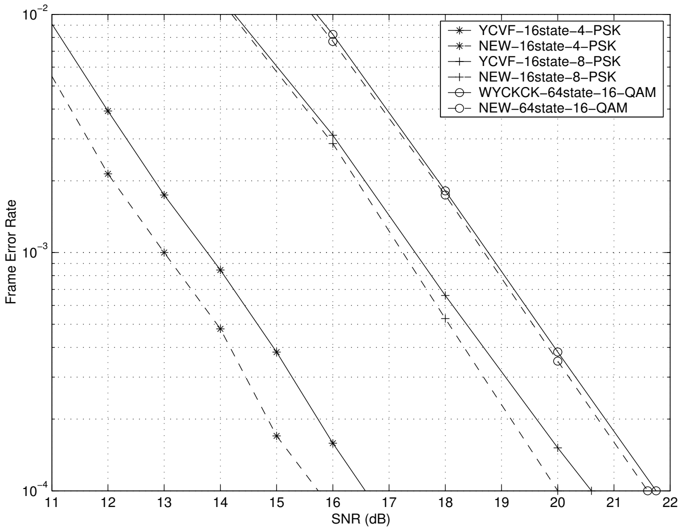


Fig. 3. Performance of 4-PSK, 8-PSK and 16-QAM STTCs, 2Tx 2Rx

outperforms the best previously known codes by 0.2 dB, at the FER of $10^{-4}$. Both the new 16 state 8-PSK code and 64 state 16-QAM code outperform the best known codes by 0.4 dB at FER of $10^{-4}$.

Fig. 3 plots the performance of 4-PSK, 8-PSK and 16-QAM STTCs with two receive antennas over slow fading channels. It is shown that the new 16 state 4-PSK and 8-PSK codes outperform the best previously known codes by 0.85 dB and 0.4 dB at the FER of $10^{-4}$. It is also shown that the new 64 state 16-QAM code outperforms the best known code by 0.16 dB at the FER of $10^{-4}$.

## V. Conclusions

Three complete sets of 4-PSK, 8-PSK and 16-QAM STTCs for two transmit antennas over slow fading channels are proposed. In order to minimize the frame error probability, the new codes are constructed by 1) guaranteeing the codeword distance matrix to be full rank, and 2) minimizing the term $(\eta(p_{max}))$ which dominated the truncated union bound. For 4-PSK and 8-PSK, new codes are found based on exhaustive search. For 16-QAM STTCs, the search for full rank codes is simplified by applying a special case of $\sum_o$ rank criterion, which save $81.5\%$ of the full code search. Through simulations, it is shown that the proposed codes outperform previously known codes for systems with low and moderate diversity gains.

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